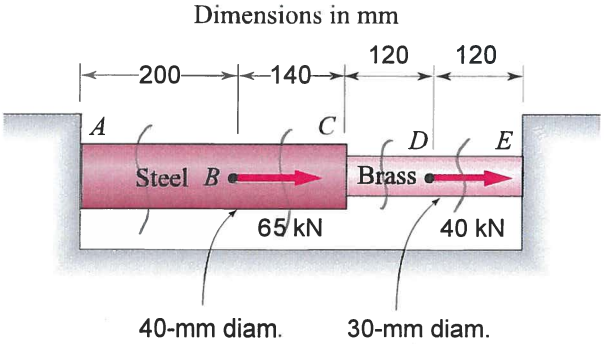
Do not round intermediate calculations. Give your final answer(s) to three significant figures.

HW3/P2

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine the deflection of point C.



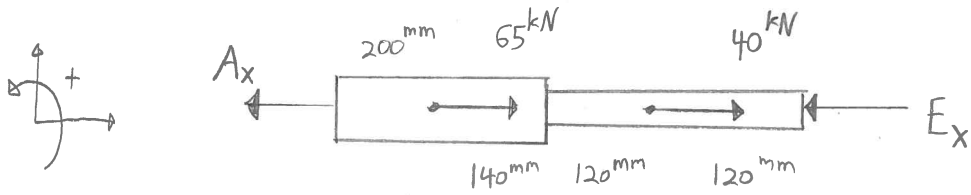
$$A_{AC} = \frac{\pi}{4}(40^{mm})^2 = 1256.64 \text{ mm}^2$$

$$A_{CE} = \frac{\pi}{4}(30^{mm})^2 = 706.86 \text{ mm}^2$$

$\delta_C =$ _____ μm →

METHOD: BRUTE FORCE

FBD$_{AE}$



INDETERMINANT ⇒ DEFORMATION RELATIONSHIP

$$\delta = \sum \frac{P_i L_i}{E_i A_i} \quad ; \quad \delta_{AE} = 0 = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$$

FBD$_{AB}$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad P = A_x$$

$$\delta_{AB} = \frac{A_x(200^{mm})}{(200E3 \text{ MPa})(1256.64 \text{ mm}^2)} = A_x(7.96E{-7})$$

FBD$_{BC}$

$$\overset{+}{\rightarrow} \sum F_x = 0; \quad P = A_x - 65^{kN}$$

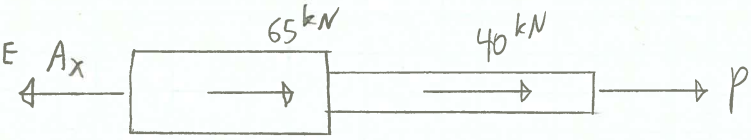
$$\delta_{BC} = \frac{(A_x - 65E3^{N})(140^{mm})}{(200E3 \text{ MPa})(1256.64 \text{ mm}^2)} = A_x(5.57E{-7}) - 3.6208E{-2}$$

FBD$_{CD}$

$$\overset{+}{\rightarrow}\sum F_x = 0; \quad P = A_x - 65^{kN}$$

$$\delta_{CD} = \frac{(A_x - 65E3)(120^{mm})}{(105E3\ MPa)(706.86\ mm^2)} = A_x(1.6168E{-6}) - 1.0509E{-1}$$

FBD$_{DE}$



$$\overset{+}{\rightarrow}\sum F_x = 0; \quad P = A_x - 105^{kN}$$

$$\delta_{DE} = \frac{(A_x - 105E3)(120^{mm})}{(105E3\ MPa)(706.86\ mm^2)} = A_x(1.6168E{-6}) - 1.6976E{-1}$$

THUS;

$$\delta_E = A_x(7.96E{-7}) + A_x(5.57E{-7}) - 3.6208E{-2}$$
$$+ A_x(1.6168E{-6}) - 1.0509E{-1} + A_x(1.6168E{-6})$$
$$- 1.6976E{-1} = 0$$

$$A_x = \frac{0.31106}{4.5866E{-6}} = 67819^{N}$$

$$\overset{+}{\rightarrow}\sum F_x = 0; \quad -67.8^{kN} + 65^{kN} + 40^{kN} - E_x = 0$$

$$\underline{\underline{E_x = 37.2\ kN}}$$

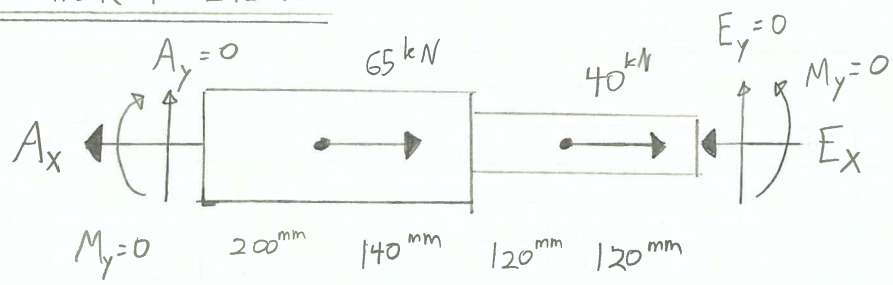
AND;

$$\delta_C = \delta_{AB} + \delta_{BC} = (67819^{N})(7.96E{-7}) + (67819^{N})(5.57E{-7}) - 3.6208E{-2}$$

$$\underline{\underline{\delta_C = 0.05555\ mm \qquad (55.5\ \mu m)}}$$

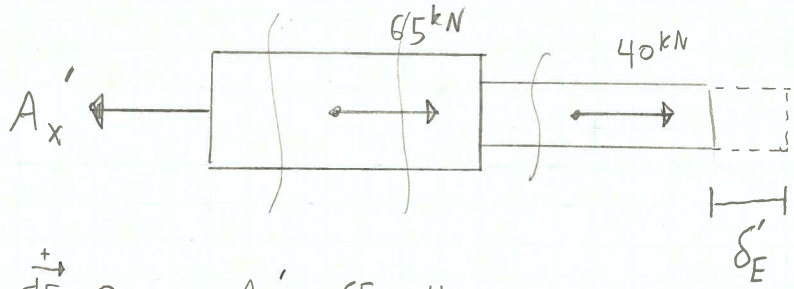
METHOD: SUPER POSITION

FBD$_{AE}$



$E_x \rightarrow$ REDUNDANT

(1)



$\overset{+}{\rightarrow} \sum F_x = 0; \quad -A_x' + 65 + 40 = 0$

$$\underline{\underline{A_x' = 105 \text{ kN}}}$$

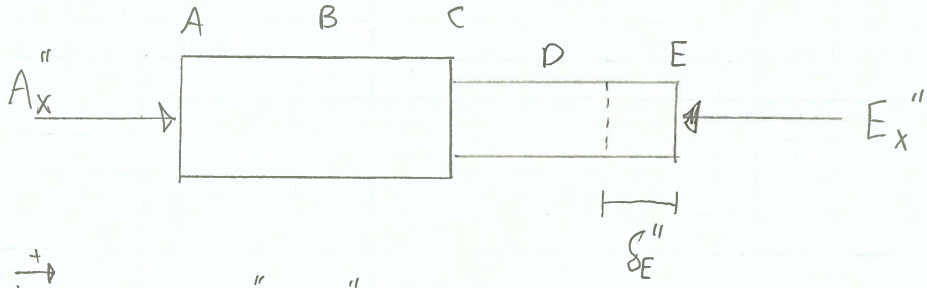
$$\delta_E' = \delta_{AB}' + \delta_{BC}' + \delta_{CD}' + \cancel{\delta_{DE}'} \rightarrow 0, \text{ No Load}$$

$$= \frac{(105E3\ N)(200\ mm)}{(1256.64\ mm^2)(200E3\ MPa)} + \frac{(40E3\ N)(140\ mm)}{(1256.64\ mm^2)(200E3\ MPa)}$$

$$+ \frac{(40E3\ N)(120\ mm)}{(706.85\ mm^2)(105E3\ MPa)}$$

$$= 0.1705 \text{ mm}$$

(2)



$\overset{+}{\rightarrow} \sum F_x = 0; \quad A_x'' = E_x''$

$$\delta_E'' = \delta_{AC}'' + \delta_{CE}''$$

$$\delta_E'' = \frac{E_x''(340^{mm})}{(1256.64\ mm^2)(200\ E3\ MPa)} + \frac{E_x''(240^{mm})}{(706.86\ mm^2)(105E3\ MPa)}$$

$$\delta_E'' = E_x''(4.5864\ E{-6})$$

(3) $$\delta_E = 0 = \delta_E' - \delta_E''$$

$$0 = 0.1705^{mm} - E_x''(4.5864\ E{-6})$$

$$E_x'' = A_x'' = \frac{0.1705}{4.5864E{-6}} = 37175^{N} \qquad (37.18^{kN})$$

THIS IS THE FORCE REQUIRED TO MAINTAIN $\delta_E = 0''$

(4) $$A_x = A_x' - A_x'' = 105^{kN} - 37.18^{kN} = 67.82\ kN$$

<u>SUMMARY</u>:

$A_x = 67.8\ kN$

$E_x = 37.2\ kN$