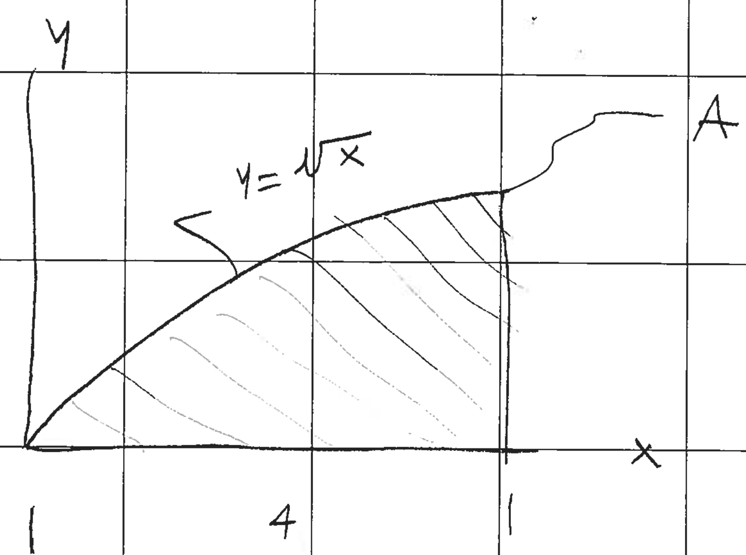
Locate the centroid of the shaded area shown



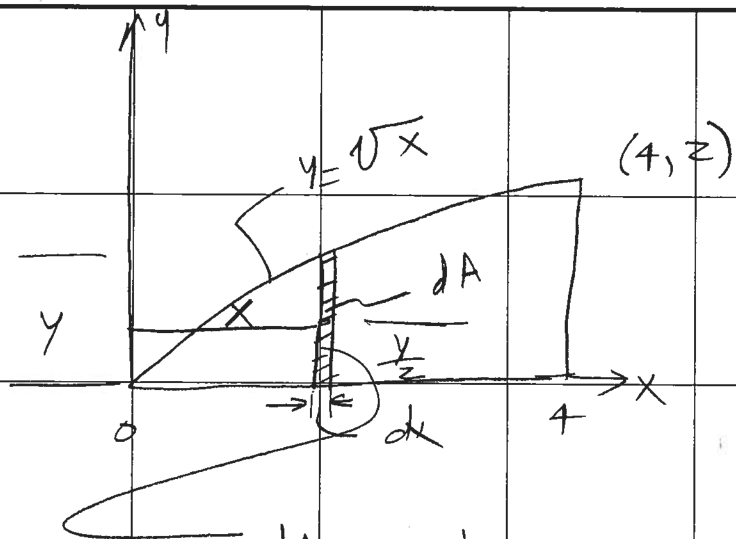
The y coordinate of point A can be found by plugging into the equation of the curve

(1) $$y=\sqrt{x}$$

(2) $$y=\sqrt{4}=2$$

So the coordinates of point A are (4, 2)

Method I: Assume a vertical differential Element as shown



(3) $dA = y\,dx$

(4) $dA = \sqrt{x}\,dx$

The area of the shaded area is

(5) $A = \int dA$

(6) $A = \int_0^4 \sqrt{x}\,dx$

(7) $\underline{A = 5.333}$ (TI-89)

(8) $M_x = \int \bar{y}\,dA$

This $\bar{y}$ is the distance from the x-axis to the centroid of the diff. element

(9) $M_x = \int_0^4 \frac{y}{2}\, dA$

(10) $M_x = \int_0^4 \frac{\sqrt{x}}{2} \sqrt{x}\, dx$

(11) $M_x = \frac{1}{2} \int_0^4 x\, dx$

(12) $M_x = 4$

(13) $\bar{Y} = \frac{M_x}{A}$

(14) $\bar{Y} = \frac{4}{5.333}$

(15) $\underline{\bar{Y} = .750}$

(16) $M_y = \int x\, dA$

This x is the distance from the y-axis to the centroid of the diff element

(17) $M_y = \int_0^4 x \sqrt{x}\, dx$

(18) $$M_y = \int_0^4 x^{\frac{3}{2}}\, dx$$

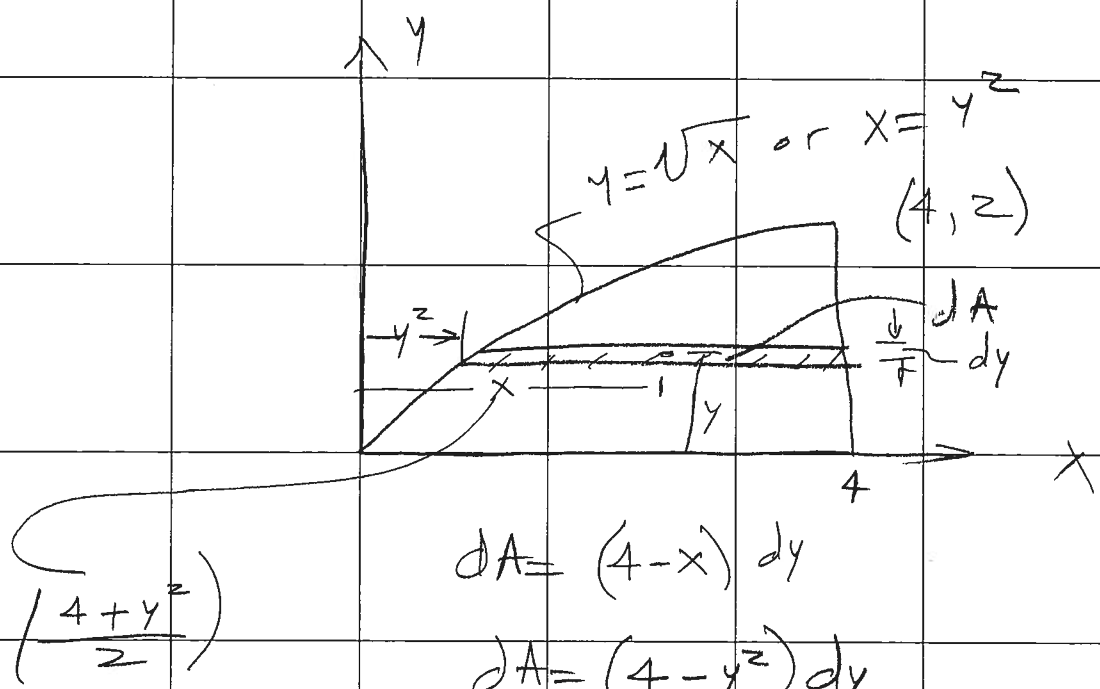
(19) $$M_y = 12.80 \quad \text{TI-89}$$

(20) $$\bar{X} = \frac{M_y}{A}$$

(21) $$\bar{X} = \frac{12.80}{5.333}$$

(22) $$\underline{\bar{X} = 2.40}$$

Method II: Assume a horizontal differential element as shown.



$$dA = (4-x)\, dy \qquad (23)$$

$$dA = (4-y^2)\, dy \qquad (24)$$

(25) $dA = \int_0^2 (4-y^2)\,dy$

(26) $A = 5.333$ TI-89

(27) $M_x = \int y\,dA$

(28) $M_x = \int_0^2 y(4-y^2)\,dy$

(29) $M_x = 4.000$ TI-89

(30) $\bar{Y} = \frac{M_x}{A}$

(31) $\bar{Y} = \frac{4.000}{5.333}$

(32) $\underline{\bar{Y} = 0.750}$

(33) $M_y = \int x\,dA$

(34) $M_y = \int_0^2 \left(\frac{4+y^2}{2}\right)(4-y^2)\,dy$

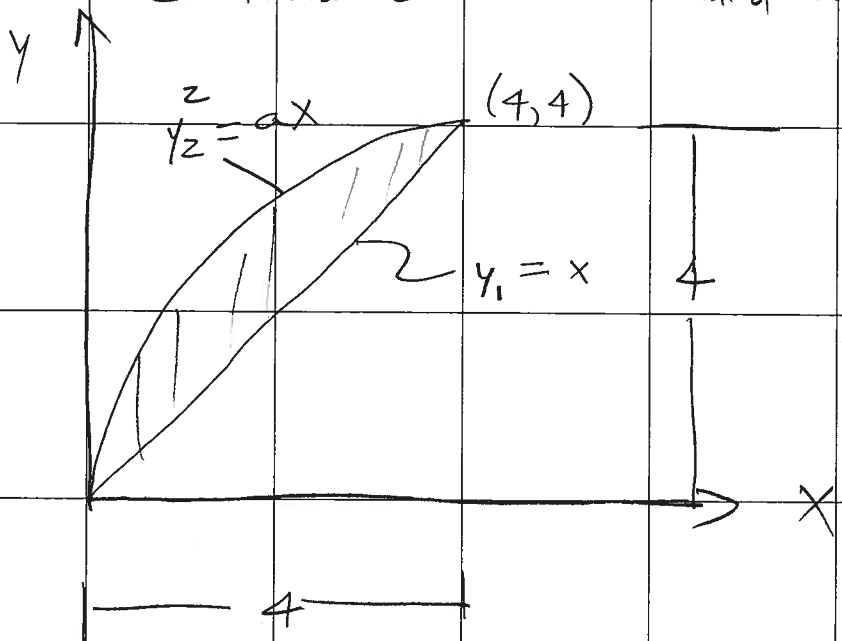
(35) $M_y = 12.8$ TI-89

(36) $\bar{X} = \frac{M_y}{A}$

(37) $\bar{X} = \frac{12.8}{5.333}$

(38) $\bar{X} = 2.40$

Locate the centroid of the shaded area shown.



The value of a can be found by checking conditions at (4,4)

$$y_2^2 = a x$$

$$(4)^2 = a(4)$$

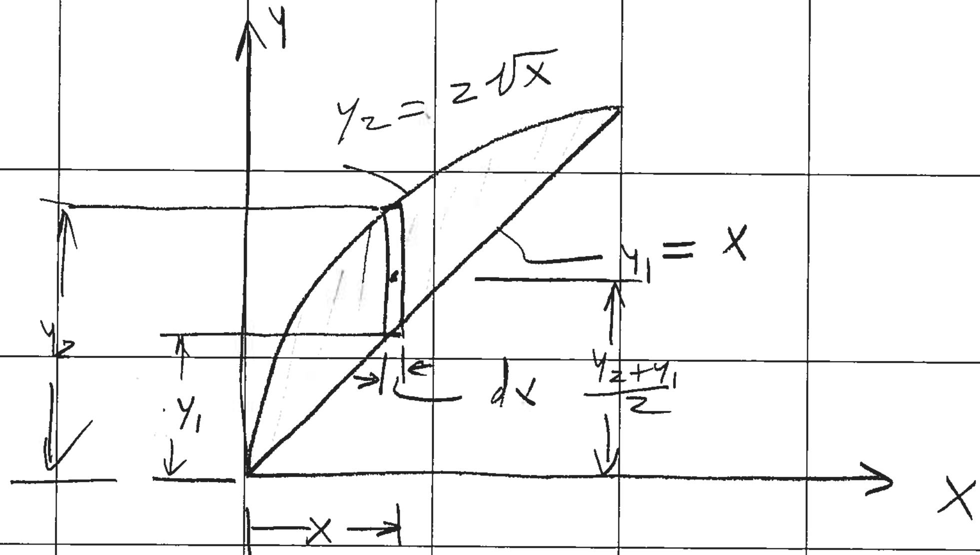
$$\underline{a = 4}$$

$$\therefore \quad y_2^2 = 4x$$

or

$$y_2 = 2\sqrt{x}$$

Method I: Use a vertical differential element as shown.



$$dA = (y_2 - y_1)\,dx$$

$$dA = (2\sqrt{x} - x)\,dx$$

$$A = \int_0^4 (2\sqrt{x} - x)\,dx$$

$\int(2*\sqrt{(x)} - x, x, 0, 4)$

$$\underline{A = 2.667}$$

$$M_x = \int y\,dA$$

$$M_x = \int_0^4 \left(\frac{y_2 + y_1}{2}\right)(y_2 - y_1)\,dx$$

$$M_x = \int_0^4 \left(\frac{2\sqrt{x} + x}{2}\right)(2\sqrt{x} - x)\,dx$$

$M_x = 5.333$ TI89

$$\bar{y} = \frac{M_x}{A}$$

$$\bar{y} = \frac{5.333}{2.667}$$

$$\underline{\bar{y} = 2.00}$$

$$M_y = \int x \, dA$$

$$M_y = \int_0^4 x(2\sqrt{x} - x)\, dx$$

$\int(x * (2 * \sqrt{}(x) - x), x, 0, 4)$

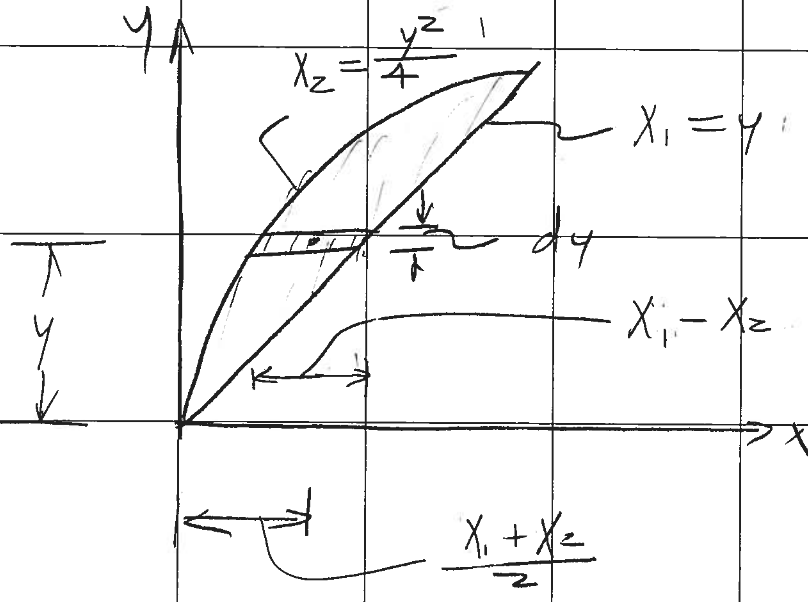
$M_y = 4.267$ TI-89

$$\bar{x} = \frac{M_y}{A}$$

$$\bar{x} = \frac{4.267}{2.667}$$

$$\underline{\bar{x} = 1.60}$$

Method II: Use a vertical differential element as shown



$$dA = (X_1 - X_2)\,dy$$

$$dA = \left(y - \frac{y^2}{4}\right) dy$$

$$A = \int_0^4 \left(y - \frac{y^2}{4}\right) dy$$

∫(y − y^2/4, y, 0, 4)

$$A = 2.667$$ TI 89

$$M_x = \int y \, dA$$

$$M_x = \int_0^4 y\left(y - \frac{y^2}{4}\right) dy$$

∫(y*(y-y^2/4),y,0,4)

$$M_x = 5.333$$

$$\bar{y} = \frac{M_x}{A}$$

$$\bar{y} = \frac{5.333}{2.667}$$

$$\underline{\bar{y} = 2.00}$$

$$M_y = \int x \, dA$$

$$M_y = \int_0^4 \left(\frac{x_1 + x_2}{2}\right)(x_1 - x_2)\, dy$$

$$M_y = \int_0^4 \left(\frac{y + \frac{y^2}{4}}{2}\right)\left(y - \frac{y^2}{4}\right) dy$$

∫(((y+y^2/4)/2)*(y-y^2/4),y,0,4)

$$M_y = 4.267$$

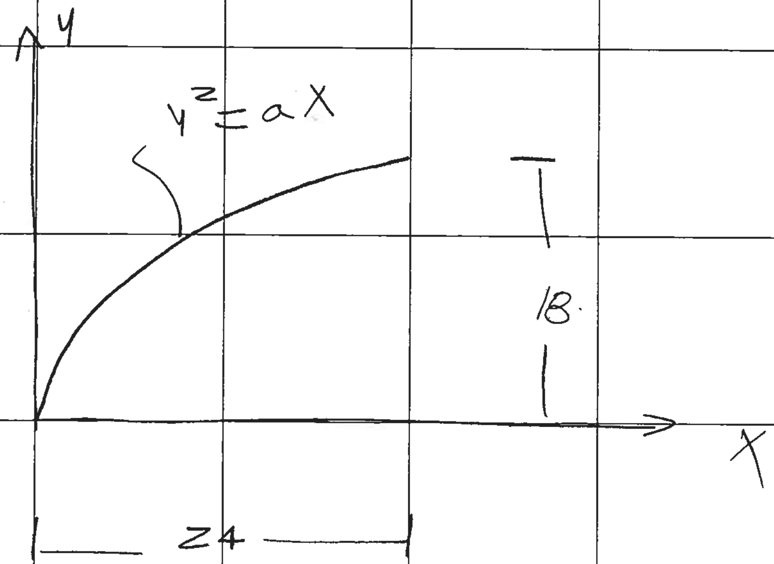
TI89

$$\bar{X} = \frac{M_y}{A}$$

$$\bar{X} = \frac{4.267}{2.667}$$

$$\underline{\bar{X} = 1.60}$$

Locate the centroid of the curved slender rod shown



First solve for a using point (24, 18)

$$18^2 = a\,(24)$$

$$a = 13.5$$

so

$$y^2 = 13.5\,X$$

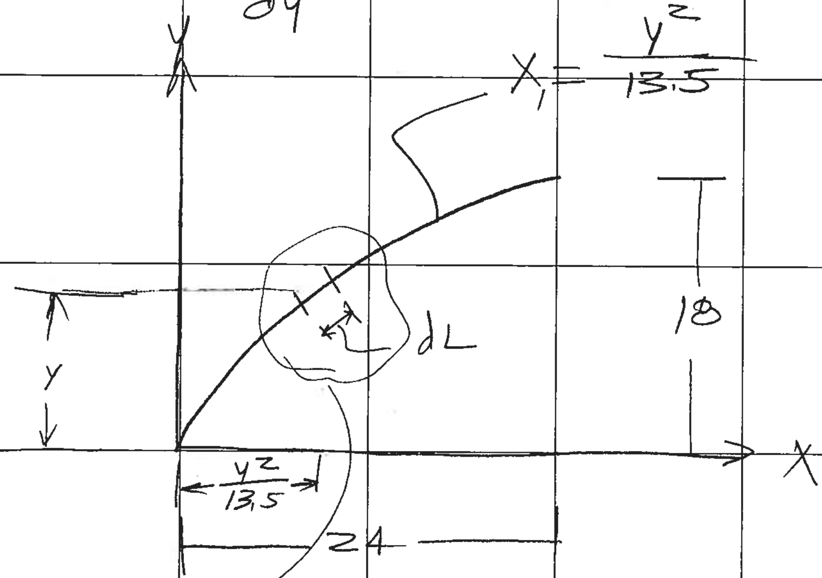
or

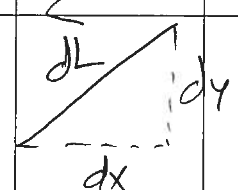
$$y = \sqrt{13.5\,X}$$

or

$$X = \frac{y^2}{13.5}$$

Method I: Integrate with respect to dy





$$dL^2 = dx^2 + dy^2$$

$$dL = \sqrt{dx^2 + dy^2}$$

$$dL = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\; dy$$

$$x = \frac{1}{13.5} y^2$$

$$\frac{dx}{dy} = \frac{2}{13.5} y$$

$$L = \int dL$$

$$L = \int_0^{18} \sqrt{\left(\frac{2}{13.5} y\right)^2 + 1}\; dy$$

$$\int\left(\sqrt{\left(\left(\boxed{(2/13.5)*y}\right)^\wedge 2+1\right)}, y, 0, 18\right)$$

$$\underline{L = 31.40}$$

$$M_x = \int y \, dL$$

$$M_x = \int_0^{18} y \sqrt{\left(\frac{2}{13.5} y\right)^2 + 1}\; dy$$

$$\int\left(y * \sqrt{\left(\left(\boxed{(2/13.5)} * y\right)^\wedge 2+1\right)}, y, 0, 18\right)$$

$$M_x = 335.65$$

$$\bar{y} = \frac{M_x}{L}$$

$$\bar{y} = \frac{335.65}{31.40}$$

$$\underline{\bar{y} = 10.69}$$

$$M_y = \int x \, dL$$

$$M_y = \int_0^{18} \frac{y^2}{13.5} \sqrt{\left(\frac{2}{13.5}y\right)^2 + 1} \; dy$$

∫(((y^2/13.5) * √(((2/13.5) * y)^2 + 1)), y, 0, 18)

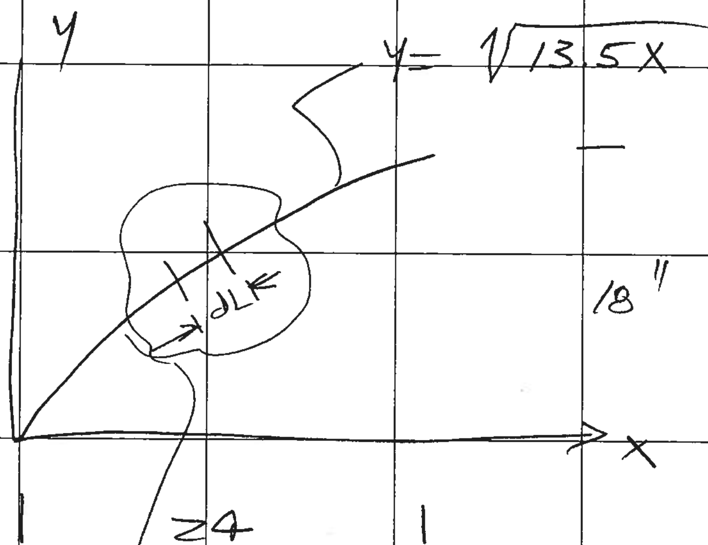
$$M_y = 324.35$$

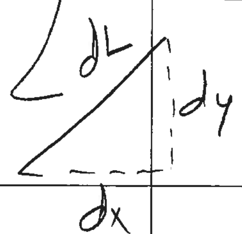
$$\bar{X} = \frac{M_y}{L}$$

$$\bar{X} = \frac{324.35}{31.40}$$

$$\underline{\bar{X} = 10.33}$$

Method II: Integrate with respect to dx





$$dL^2 = dx^2 + dy^2$$

$$dL = \sqrt{dx^2 + dy^2}$$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y = \sqrt{13.5}\quad x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \left(\frac{1}{2}\right)\sqrt{13.5}\quad x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{\sqrt{\frac{13.5}{4}}}{\sqrt{x}} = \sqrt{\frac{13.5}{4x}}$$

$$L = \int dL$$

$$L = \int_0^{24} \sqrt{1 + \left(\frac{13.5}{4x}\right)}\, dx$$

∫(√(1+ 13.5/(4* X)), X, 0, 24)

$$L = 31.40$$

$$M_x = \int y\, dL$$

$$M_x = \int_0^{24} \sqrt{13.5x}\ \sqrt{1 + \left(\frac{13.5}{4x}\right)}\, dx$$

∫(√(13.5*X) * √(1+ 13.5/(4*X)), X, 0, 24)

$$M_x = 335.65$$

$$\bar{y} = \frac{M_x}{L}$$

$$\bar{y} = \frac{335.65}{31.40}$$

$$\underline{\bar{y} = 10.69}$$

$$M_y = \int x \, dL$$

$$M_y = \int_0^{24} x \sqrt{1 + \left(\frac{13.5}{4x}\right)} \, dx$$

$$\int\left(x * \sqrt{\left(1 + 13.5/(4 * x)\right)}, x, 0, 24\right)$$

$$M_y = 324.35$$

$$\bar{X} = \frac{M_y}{L}$$

$$\bar{X} = \frac{324.35}{31.40}$$

$$\bar{x} = 10.33$$