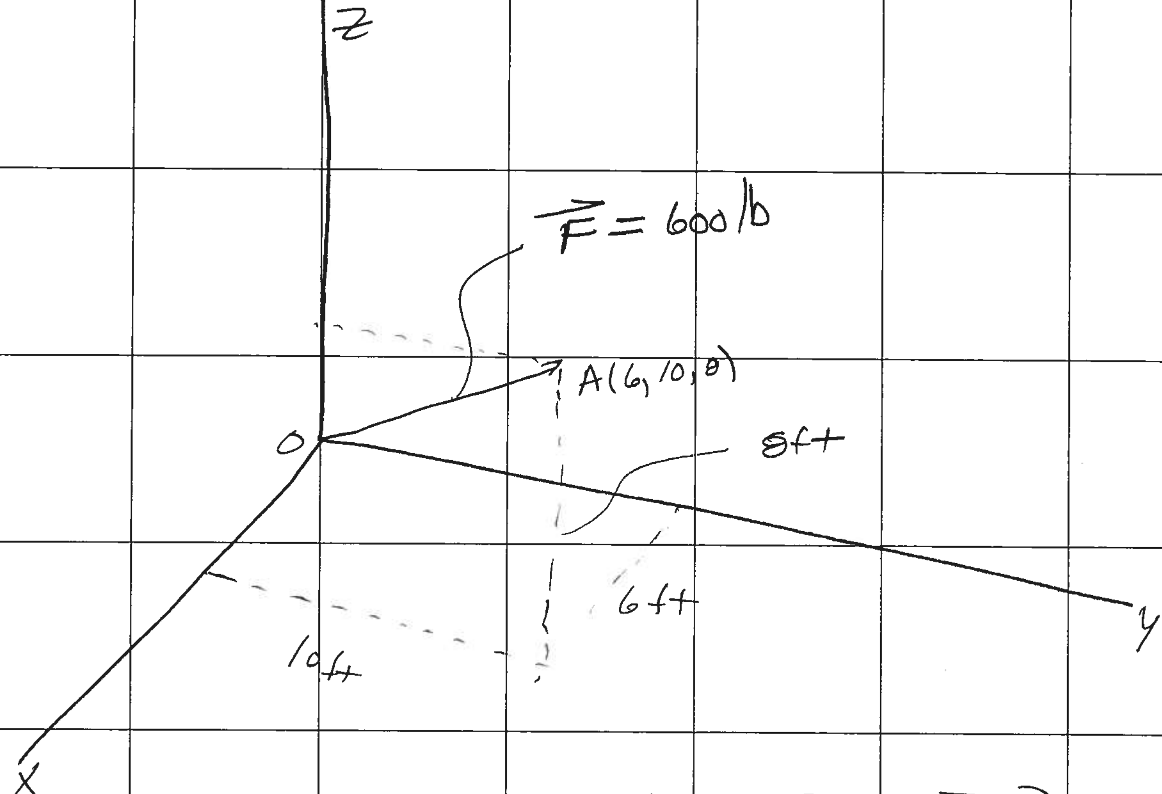
Determine the x, y, and z components of the force shown and express the force in Cartesian vector form.



The position vector from $\vec{OA}$ is given as follows

(1) $$\vec{OA} = 6\hat{i} + 10\hat{j} + 8\hat{k} \quad (ft)$$

(2) $$|\vec{OA}| = \sqrt{6^2 + 10^2 + 8^2}$$

(3) $$|\vec{OA}| = 14.14\ ft$$

A unit vector in the direction of $\vec{OA}$ is given as follows

(4) $\hat{n}_{OA} = \frac{6}{14.14}\hat{i} + \frac{10}{14.14}\hat{j} + \frac{8}{14.14}\hat{k}$

(5) $\hat{n}_{OA} = .424\,\hat{i} + .707\,\hat{j} + .566\,\hat{k}$

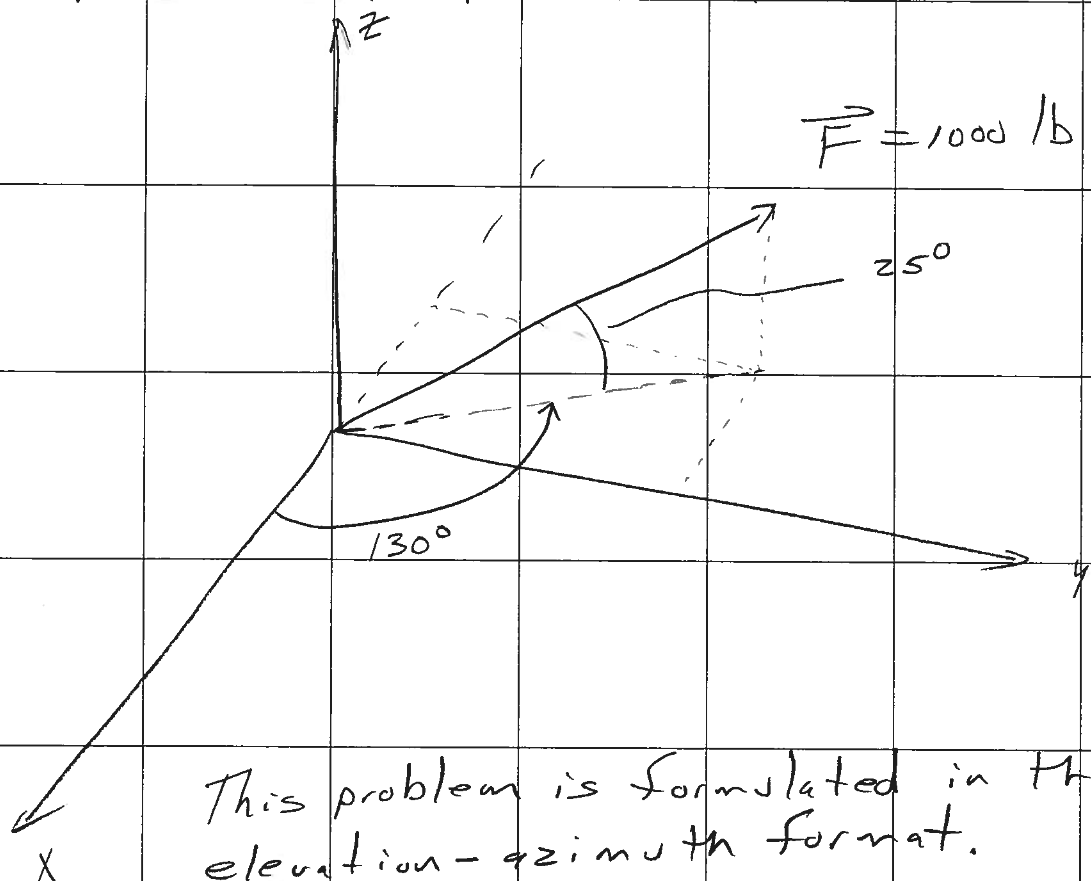
Then the force $\vec{F}$ is given as follows

(6) $\vec{F} = F\,\hat{n}_{OA}$

(7) $\vec{F} = 600(.424)\,\hat{i} + 600(.707)\,\hat{j} + 600(.566)\,\hat{k}$ lbs

(8) $\vec{F} = 254.4\,\hat{i} + 424.2\,\hat{j} + 339.6\,\hat{k}$ lbs

Determine the x, y, and z components of the force shown and express force in Cartesian vector form.



This problem is formulated in the elevation-azimuth format.

The best way is to use e-a method as a guide and establish the signs by inspection.

It is clear by inspection that F_x is ⊖, F_y is ⊕, and F_z is ⊕

Then start with F_z

$$F_z = +\sin 25° \,(1000\text{ lb})$$

(1)

(2) $\underline{F_z = 422.6 \text{ lb}}$

Then F_{xy} is found as follows

(3) $F_{xy} = \cos 25° (1000 \text{ lb})$

(4) $\underline{F_{xy} = 906.3 \text{ lb}}$

$130° - 90° = 40°$

Then by inspection

(5) $F_x = -\sin 40° (906.3 \text{ lb})$

(6) $\underline{F_x = -582.6 \text{ lb}}$

and

(7) $F_y = +\cos 40° (906.3 \text{ lb})$

(8) $\underline{F_y = 694.3 \text{ lb}}$

OR

The problem can be solved with the following equations

(9) $F_x = F \cos\phi \cos\theta$

(10) $F_y = F \cos\phi \sin\theta$

(11) $F_z = F \sin\phi$

where θ is the azimuth angle measured ⊕ CCW from the X-axis and ϕ is the elevation angle measured ⊕ above the X-Y plane.

Thus

(12) $\theta = 130°$ and $\phi = 25°$

Then

(13) $F_x = (1000\ lb) \cos 25° \cos 130°$

(14) $F_x = -582.6\ lbs$

(15) $F_y = (1000\ lb) \cos 25° \sin 130°$

(16) $F_y = 694.3\ lbs$

(17) $F_z = (1000\ lb) \sin 25°$

(18) $F_z = 422.6\ lbs$

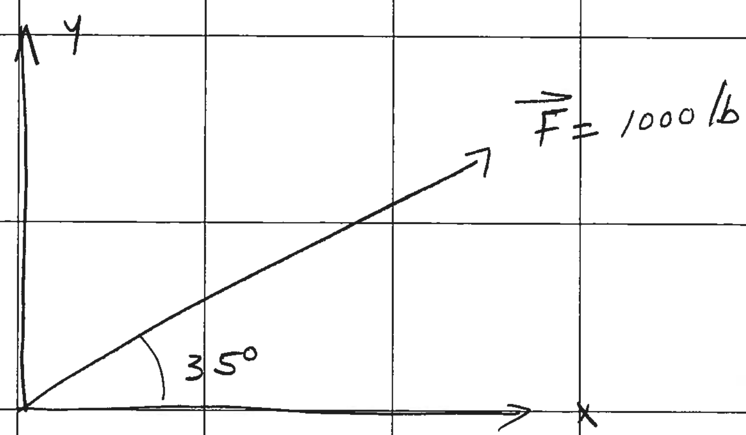
(19) $\vec{F} = -582.6\ \hat{i} + 694.3\ \hat{j} + 422.6\ \hat{k}$ (1b)

W. Lynn Beason, Ph.D., P.E.
Statics

Topic Concurrent Forces

Determine the x and y scalar components of the forces shown. Express in Cartesian vector form

a)



(1) $F_x = \cos 35° (1000\ lb)$

(2) $\underline{F_x = 819.2\ lb}$

(3) $F_y = \sin 35° (1000\ lb)$

(4) $F_y = 573.6\ lb$

(5) $F_z = 0$

(6) $\vec{F} = 819.2\ \hat{i} + 573.6\ \hat{j} + 0\ \hat{k}$ (1b)

By inspection

(13) $F_x = +\cos 30\ (500\ lb)$

(14) $F_x = 433.0\ lb$

By inspection

(15) $F_y = -\sin 30\ (500\ lb)$

(16) $\underline{F_y = -250\ lb}$

(17) $F_z = 0$

or

θ is measured CCW from x-axis

(18) $\therefore\ \theta = 330^0$ or -30^0

(19) $F_x = \cos 330^0\ (500\ lb)$

(20) $\underline{F_x = 433.0\ lb}$

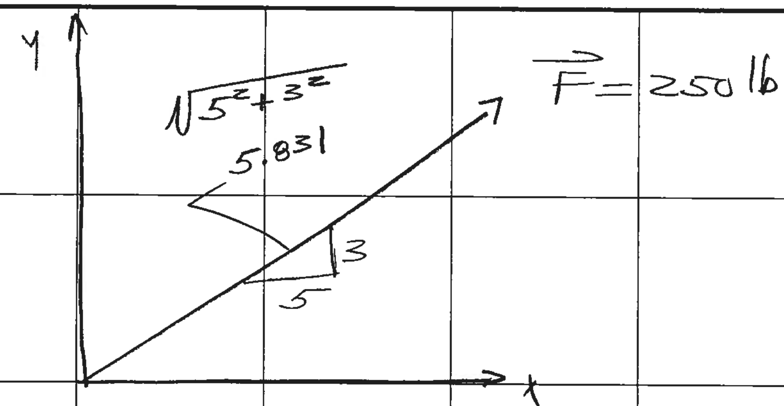
(21) $F_y = \sin 330^0 (500\ lb)$

(22) $\underline{F_y = -250\ lb}$

(23) $F_z = 0$

(24) $\vec{F} = 433\ \hat{i} - 250\ \hat{j} + 0\ \hat{k}$

b)



(7) $F_x = \frac{5}{5.831}(250 \text{ lb})$

(8) $\underline{F_x = 214.4 \text{ lb}}$

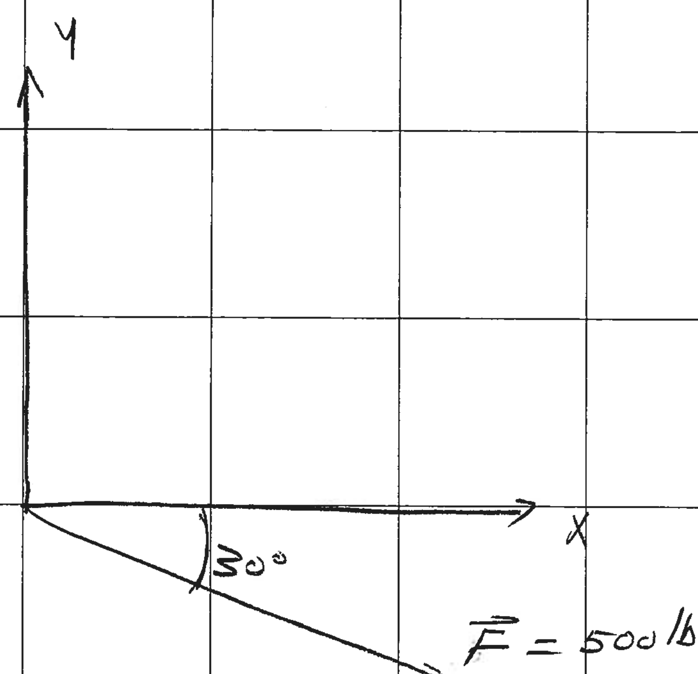
(9) $F_y = \frac{3}{5.831}(250 \text{ lb})$

(10) $\underline{F_y = 128.6 \text{ lb}}$

(11) $F_z = 0$

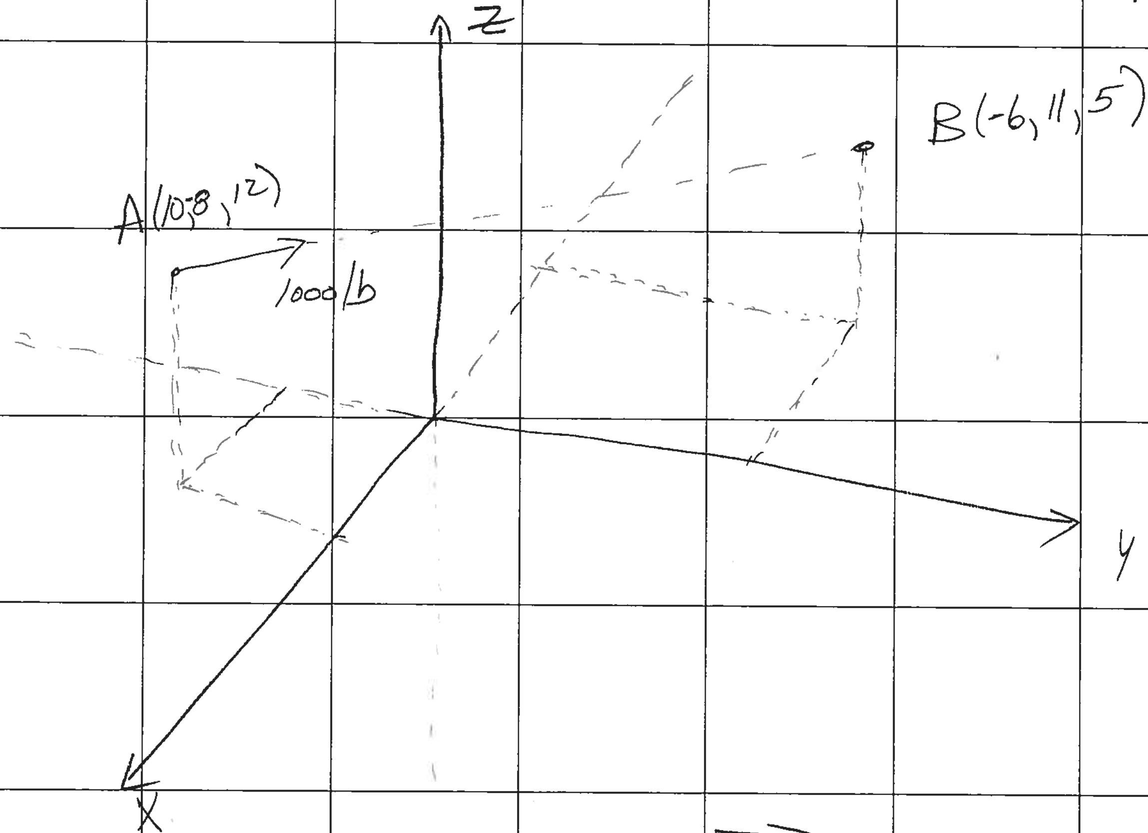
(12) $\vec{F} = 214.4\,\hat{i} + 128.6\,\hat{j} + 0\,\hat{k}$ (lb)

c)



W. Lynn Beason, Ph.D., P.E.
Statics

A force of 1000 lb is applied on a line from point A to point B. Express this force in Cartesian vector form. Also determine the angles θ_x, θ_y, and θ_z between the force and the positive x, y, and z directions, respectively.



First write vector $\overrightarrow{AB}$

(1) $\overrightarrow{AB} = (-6-10)\,\hat{i} + (11-(-8))\,\hat{j} + (5-12)\,\hat{k}$

(2) $\overrightarrow{AB} = -16\,\hat{i} + 19\,\hat{j} - 7\,\hat{k}$

Then calculate the magnitude of $\overrightarrow{AB}$

(3) $|\overrightarrow{AB}| = \sqrt{(-16)^2 + 19^2 + (-7)^2}$

(4) $|\overrightarrow{AB}| = 25.81$

Next find a unit vector pointing in direction of $\overrightarrow{AB}$ by normalizing AB as follows.

(5) $\hat{n}_{AB} = -\frac{16}{25.81}\hat{i} + \frac{19}{25.81}\hat{j} - \frac{7}{25.81}\hat{k}$

(6) $\hat{n}_{AB} = -.620\,\hat{i} + .736\,\hat{j} - .271\,\hat{k}$

Then the force is determined by multiplying the magnitude of the force by the unit vector, $\hat{n}_{AB}$

(7) $\overrightarrow{F}_{AB} = 1000\left(-.620\,\hat{i} + .736\,\hat{j} - .271\,\hat{k}\right)$

(8) $\overrightarrow{F}_{AB} = -620\,\hat{i} + 736\,\hat{j} - 271\,\hat{k}$ (lb)

The direction cosines are the components of the unit vector, $\hat{n}_{AB}$

(9) $\cos \theta_x = -.620$

(10) $\theta_x = 128.3^\circ$

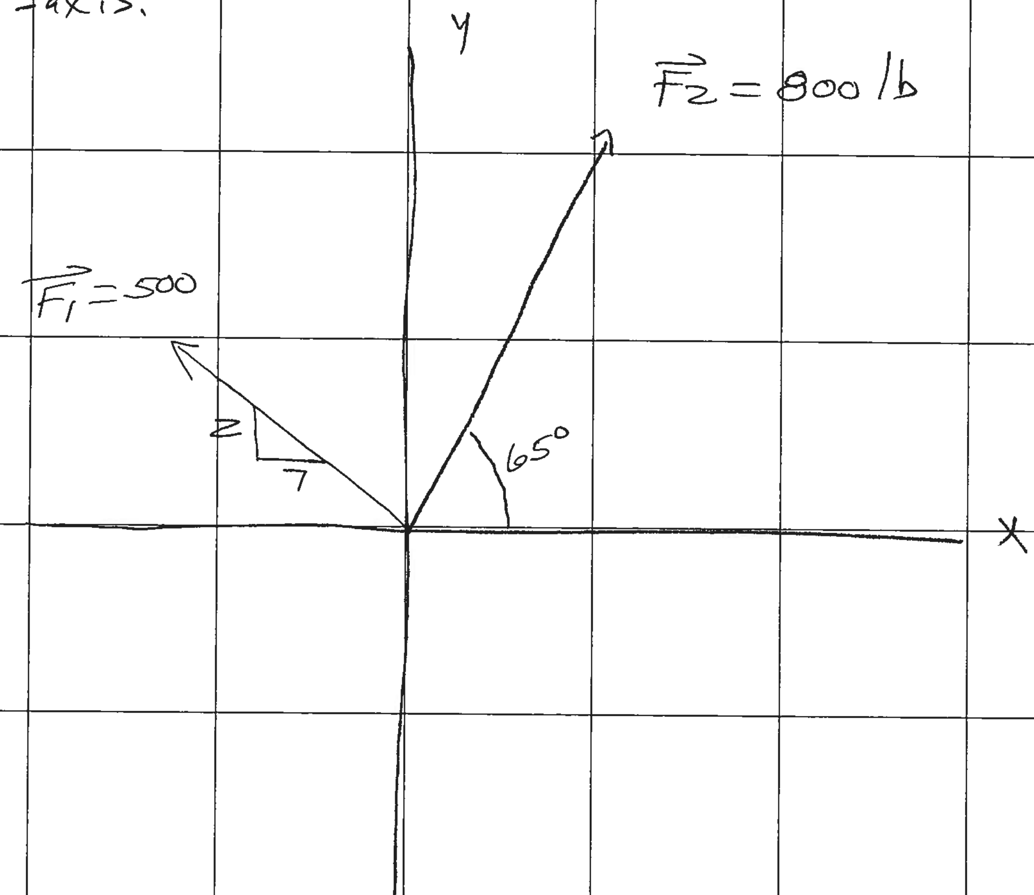
(11) $\cos \theta_y = .736$

(12) $\theta_y = 42.6^\circ$

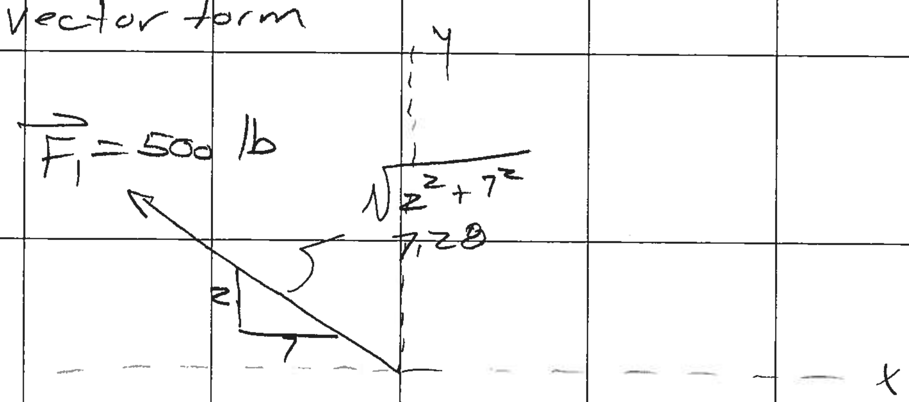
(13) $\cos \theta_z = -.271$

(14) $\theta_z = 105.7^\circ$

Determine the resultant of the forces shown and the angles θ_x and θ_y between the line of action of the resultant and the x-axis and y-axis.



First express force $\vec{F_1}$ in a Cartesian vector form

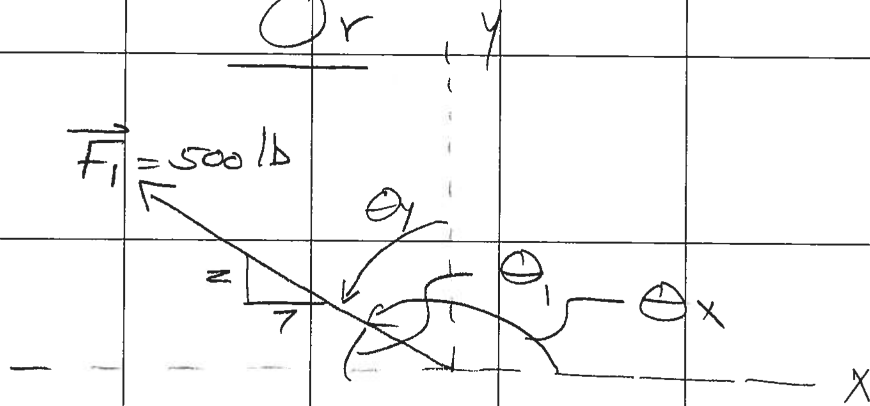


Then by inspection

(1) $\vec{F}_1 = -\frac{7}{7.28}(500)\hat{i} + \frac{2}{7.28}\hat{j} + 0\hat{k}$ (lb)

(2) $\vec{F}_1 = -480.8\hat{i} + 137.4\hat{j} + 0\hat{k}$ (lb)

Or



(3) $\theta_1 = \tan^{-1}\frac{2}{7}$

(4) $\theta_1 = 15.95^\circ$

This means

(5) $\theta_x = 180^\circ - 15.95^\circ = 164.05^\circ$

(6) $\theta_y = 90^\circ - 15.95^\circ = 74.05^\circ$

Because this is a 2-D problem

(7) $\theta_z = 90^\circ$

(8) $$\vec{F}_1 = (\cos 164.05^\circ)(500)\,\hat{i} + (\cos 74.05^\circ)(500)\,\hat{j} + (\cos 90^\circ)(500)\,\hat{k}$$

(9) $$\underline{\vec{F}_1 = -480.8\,\hat{i} + 137.4\,\hat{j} + 0\,\hat{k}}$$

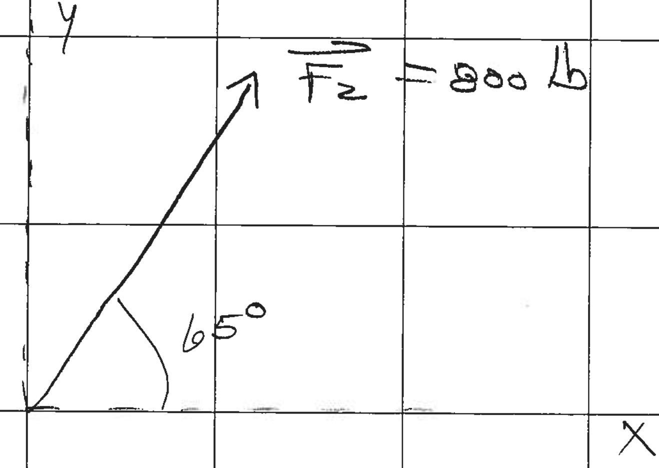
<u>Or</u>

By inspection

(10) $$\vec{F}_1 = -\cos 15.95^\circ\,(500)\,\hat{i} + \sin 15.95^\circ\,(500)\,\hat{j} + 0\,\hat{k}$$

(11) $$\underline{\vec{F}_1 = -480.8\,\hat{i} + 137.4\,\hat{j} + 0\,\hat{k}}$$

Next express force $\vec{F_2}$ in Cartesian vector form



By inspection

(12) $$\vec{F_2} = (\cos 65^\circ)(800)\,\hat{i} + (\sin 65^\circ)(800)\,\hat{j} + 0\,\hat{k}$$

(13) $$\underline{\vec{F_2} = 338.1\,\hat{i} + 725.1\,\hat{j} + 0\,\hat{k}}$$

Then simply add the components of the two forces to determine the resultant

(14) $$\vec{R} = \vec{F_1} + \vec{F_2}$$

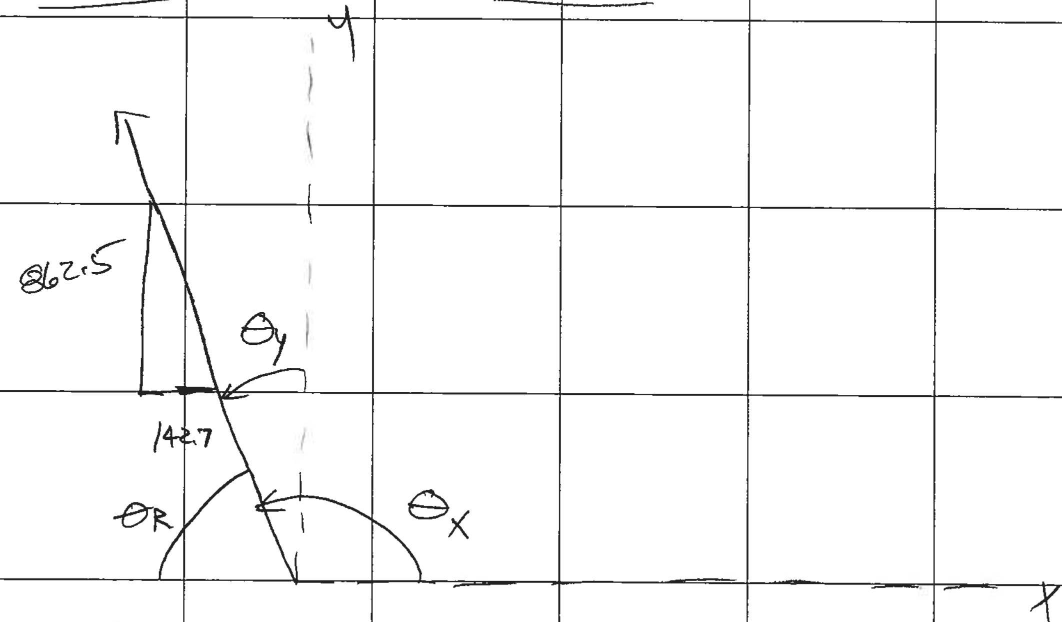
(15) $\vec{R} = (-480.8 + 338.1)\,\hat{i} + (137.4 + 725.1)\,\hat{j} + (0 + 0)\,\hat{k}$

(16) $\vec{R} = -142.7\,\hat{i} + 862.5\,\hat{j} + 0\,\hat{k}$

The magnitude of the resultant is

(17) $|\vec{R}| = \sqrt{(-142.7)^2 + (862.5)^2 + 0^2}$

(18) $|\vec{R}| = 874.2 \text{ lb}$



By inspection

(19) $\theta_R = \mathrm{Tan}^{-1} \frac{862.5}{142.7}$

(20) $\theta_R = 80.6^\circ$

Then

(21) $\theta_x = 180^\circ - 80.6^\circ = 99.4^\circ$

(22) $\theta_y = 90^\circ - 80.6^\circ = 9.4^\circ$

<u>Or</u>

Normalize $\vec{R}$ as follows

(23) $\hat{n}_R = -\frac{142.7}{874.2}\hat{i} + \frac{862.5}{874.2}\hat{j} + 0\hat{k}$

(24) $\hat{n}_R = -.163\,\hat{i} + .987\,\hat{j} + 0\hat{k}$

(25) $\theta_x = \cos^{-1}(-.163)$

(26) $\theta_x = 99.4^\circ$

(27) $\theta_y = \cos^{-1}(.987)$

(28) $\theta_y = 9.3^\circ \simeq 9.4^\circ$

of course because this is a 2-D problem

(2a) $\theta_z = \cos^{-1}(0)$

(3c) $\underline{\theta_z = 0}$