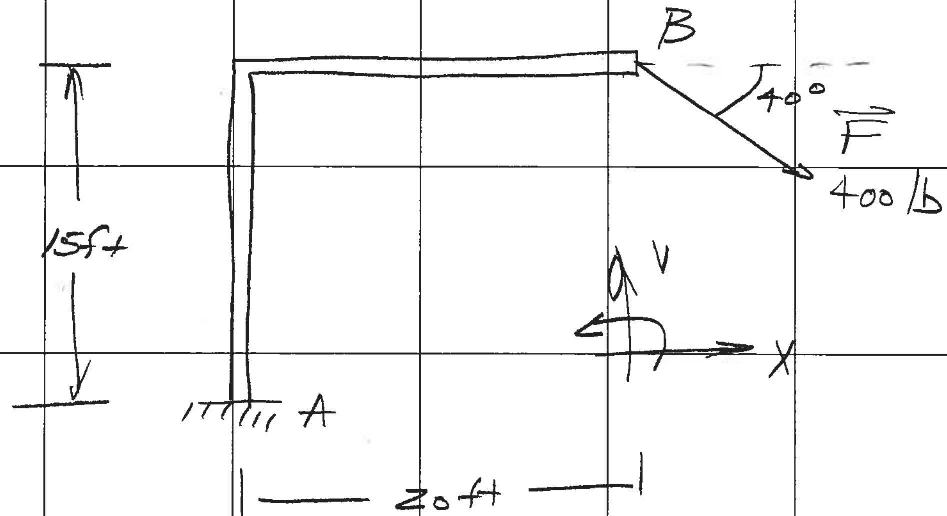
Determine the moment of the 400 lb force about point A.



Method I $\vec{r} \times \vec{F}$

Express the 400 lb force as a vector by inspection

(1) $\vec{F} = \cos 40^\circ (400)\hat{i} - \sin 40^\circ (400)\hat{j} + 0\hat{k}$

(2) $\vec{F} = 306.4\hat{i} - 257.1\hat{j} + 0\hat{k}$

Express $\vec{r}$ vector by inspection

(3) $\vec{r}_{AB} = 20\hat{i} + 15\hat{j} + 0\hat{k}$

The force is applied at B and we are taking moment about point A, hence we use $\vec{r}_{AB}$ and not $\vec{r}_{BA}$

Evaluate $\vec{r}_{AB} \times \vec{F}$

(4)

$$\vec{r}_{AB} \times \vec{F} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ 20 & 15 & 0 \\ 306.4 & -257.1 & 0 \end{vmatrix}$$

(15)

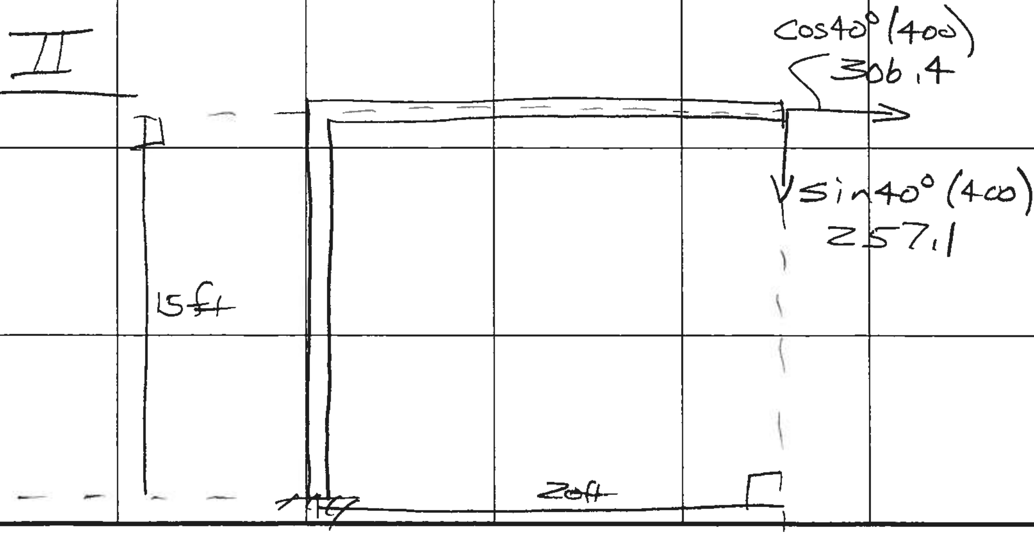
$$= +\left[15(0) - (-257.1)(0)\right]\hat{i}$$

$$- \left[20(0) - 306.4(0)\right]\hat{j}$$

$$+ \left[(20)(-257.1) - (306.4)(15)\right]\hat{k}$$

(16) $\vec{r}_{AB} \times \vec{F} = 0\,\hat{i} + 0\,\hat{j} - 9738\,\hat{k}$ ft-lb

Method II



There are no moments about the x and y axes because this is a 2-D problem.

z-moment by inspection

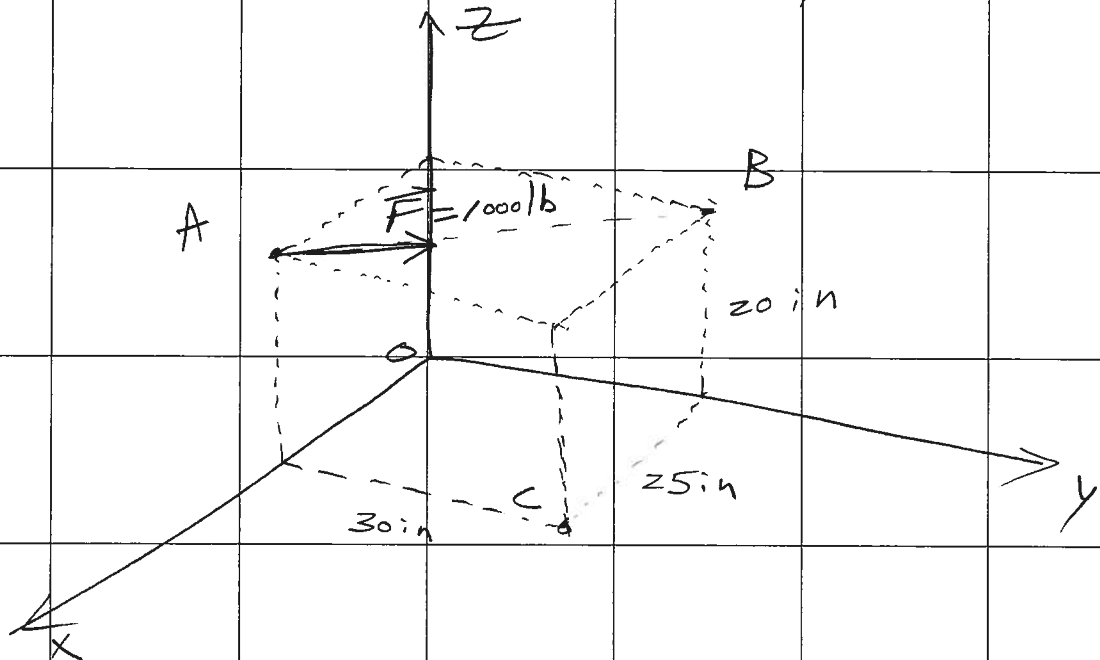
(17) $M_z = -(306.4)(15) - (257.1)(20)$

(18) $M_z = -9738$ ft-lb +

or

+ 9738 ft-lb

A force of 1000 lb is applied to a point in a body as shown. Determine the moment of the force about point C



First - express $\vec{F}$ as a vector

The coordinates of A are (25, 0, 20)

The coordinates of B are (0, 30, 20)

(1) $\overrightarrow{AB} = (0-25)\hat{i} + (30-0)\hat{j} + (20-20)\hat{k}$

(2) $\overrightarrow{AB} = -25\hat{i} + 30\hat{j} + 0\hat{k}$

(3) $|\overrightarrow{AB}| = \sqrt{(-25)^2 + (30)^2 + (0)^2} = 39.05$

(4) $\hat{\lambda}_{AB} = -\frac{25}{39.05}\hat{i} + \frac{30}{39.05}\hat{j} + 0\hat{k}$

(5) $\hat{n}_{AB} = -.640\,\hat{i} + .768\,\hat{j} + 0\,\hat{k}$

(6) $\vec{F}_{AB} = 1000\left[-.640\,\hat{i} + .768\,\hat{j} + 0\,\hat{k}\right]$

(7) $\vec{F}_{AB} = -640\,\hat{i} + 768\,\hat{j} + 0\,\hat{k}$

Method I calculate moment using $\vec{r}_{CA}$

by inspection

(8) $\vec{r}_{CA} = 0\,\hat{i} - 30\,\hat{j} + 20\,\hat{k}$

(9) $\vec{M}_{F\,about\,C} = \vec{r}_{CA} \times \vec{F}$

(10) $\vec{r}_{CA} \times \vec{F} \quad \begin{array}{ccc} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \end{array}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -30 & 20 \\ -640 & 768 & 0 \end{vmatrix}$$

(11) $= +\left[(-30)(0) - (768)(20)\right]\hat{i}$

$-\left[(0)(0) - (-640)(20)\right]\hat{j}$

$+\left[0(768) - (-640)(-30)\right]\hat{k}$

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(12) $\vec{r}_{CA} \times \vec{F} = -15360\,\hat{i} - 12{,}800\,\hat{j} - 19{,}200\,\hat{k}$ (in-lb)

the magnitude of the moment is

(13) $\left| \vec{M}_{\vec{F}\,about\,C} \right| = \sqrt{(-15360)^2 + (-12{,}800)^2 + (-19{,}200)^2}$

(14) $= 27{,}720.2$ in-lb

Method II calculate moment using $\vec{r}_{CB}$

by inspection

(15) $\vec{r}_{CB} = -25\,\hat{i} + 0\,\hat{j} + 20\,\hat{k}$

$\vec{M}_{\vec{F}\,about\,C} = \vec{r}_{CB} \times \vec{F}$ (16)

$$\vec{r}_{CB} \times \vec{F} = \begin{vmatrix} \overset{+}{\hat{i}} & \overset{-}{\hat{j}} & \overset{+}{\hat{k}} \\ -25 & 0 & 20 \\ -640 & 768 & 0 \end{vmatrix}$$ (17)

$$= +\left[(0)(0) - (768)(20)\right]\hat{i} \qquad (18)$$

$$-\left[(-25)(0) - (-640)(20)\right]\hat{j}$$

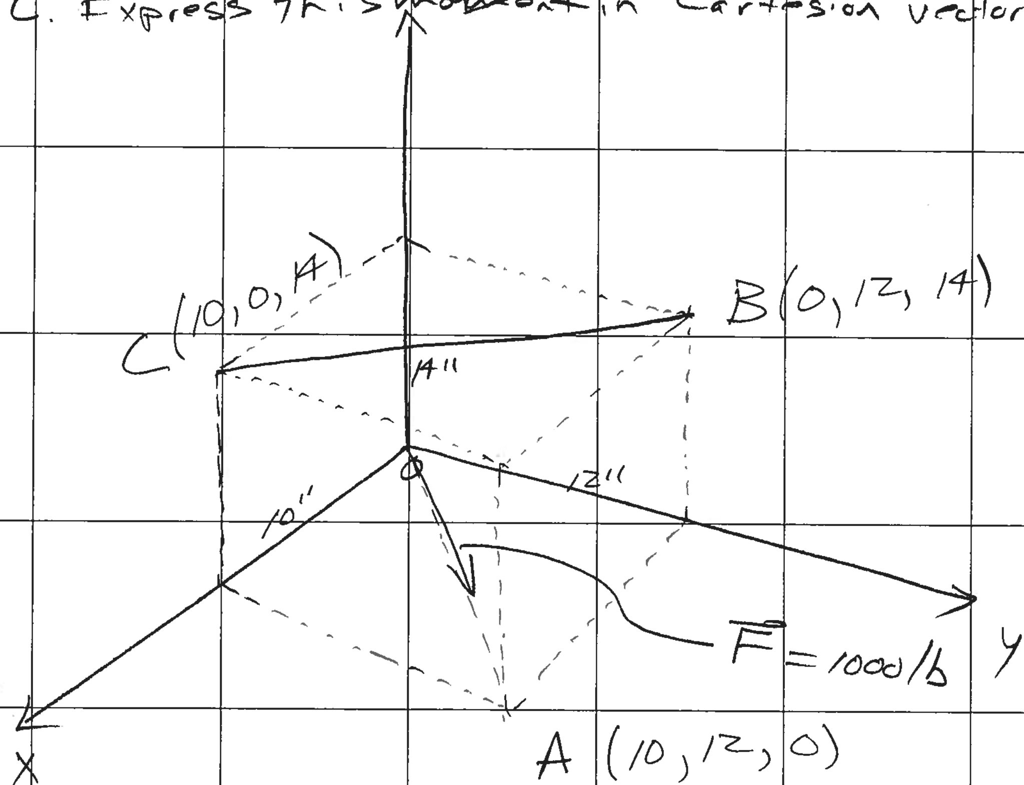
$$+\left[(-25)(768) - (-640)(0)\right]\hat{k}$$

(19)

$$\vec{r}_{CB} \times \vec{F} = -15360\,\hat{i} - 12{,}800\,\hat{j} - 19{,}200\,\hat{k}$$

This is the same result as the other method.

A force of 1000 lb is applied along a line from $\overrightarrow{OA}$. Determine the scalar component of the moment about line $\overrightarrow{BC}$. Express this moment in Cartesian vector components



First express $\overline{F}$ as a vector by inspection

(1) $\overrightarrow{OA} = 10\,\hat{i} + 12\,\hat{j} + 0\,\hat{k}$

(2) $|\overrightarrow{OA}| = \sqrt{10^2 + 12^2 + 0^2} = 15.62$

(3) $\hat{n}_{OA} = \frac{10}{15.62}\,\hat{i} + \frac{12}{15.62}\,\hat{j} + 0\,\hat{k}$

(4) $\hat{n}_{OA} = .640\,\hat{i} + .768\,\hat{j} + 0\,\hat{k}$

(5) $\vec{F} = .640(1000)\,\hat{i} + .768\,(1000)\,\hat{j} + 0\,\hat{k}$

(6) $\vec{F} = 640\,\hat{i} + 768\,\hat{j} + 0\hat{k}$

Next express $\overrightarrow{BC}$ as a vector

(7) $\overrightarrow{BC} = (10-0)\,\hat{i} + (0-12)\,\hat{j} + (14-14)\,\hat{k}$

(8) $\overrightarrow{BC} = 10\,\hat{i} - 12\,\hat{j} + 0\,\hat{k}$

(9) $|\overrightarrow{BC}| = \sqrt{10^2 + (-12)^2 + 0^2} = 15.62$

Next express a vector that connects any point on line BC with any point on the line of action of the force ($\overrightarrow{OA}$)

I will use $\overrightarrow{BO}$

by inspection

(10) $\overrightarrow{BO} = 0\,\hat{i} + 12\hat{j} + 14\hat{k}$

Then

(11) $M_{\vec{F}\text{ about }\overrightarrow{BC}} = \left(\overrightarrow{BO} \times \vec{F}\right) \cdot \hat{n}_{BC}$

Where $\hat{n}_{BC}$ is a unit vector in the direction of $\overrightarrow{BC}$ which is given as follows

(12) $\hat{n}_{BC} = \frac{10}{15.62}\hat{i} - \frac{12}{15.62}\hat{j} + 0\hat{k}$

(13) $\hat{n}_{BC} = .640\hat{i} - .768\hat{j} + 0\hat{k}$

Then

$$(\vec{BO} \times \vec{F}) \cdot \hat{n}_{BC} = \begin{array}{ccc} + & - & + \\ \end{array}\begin{vmatrix} .640 & -.768 & 0 \\ 0 & 12 & 14 \\ 640 & 768 & 0 \end{vmatrix} \quad (14)$$

$$= +.640\left[(12)(0) - 768(14)\right] \quad (15)$$
$$- (-.768)\left[.640(0) - 640(0)\right]$$
$$+ (0)\left[0(768) - (640)(12)\right]$$

$$\boxed{(\vec{BO} \times \vec{F}) \cdot \hat{n}_{BC} = -6881.3 \text{ in-lb}} \quad (16)$$

The components of this moment are found by multiplying this magnitude by $\hat{n}_{BC}$ as follows

(17) $-6881.3 \ \hat{n}_{BC}$

(18) $-6881.3 \left(.640\,\hat{i} - .768\,\hat{j} + 0\,\hat{k} \right)$

(19) $\boxed{-4404\,\hat{i} + 5285\,\hat{j} + 0\,\hat{k}}$