STATICS

FORCE

A *force* is a *vector* quantity. It is defined when its **(1)** magnitude, **(2)** point of application, and **(3)** direction are known.

RESULTANT (TWO DIMENSIONS)

The *resultant*, F, of n forces with components $F_{x,i}$ and $F_{y,i}$ has the magnitude of

$$F = \left[\left(\sum_{i=1}^{n} F_{x,i}\right)^2 + \left(\sum_{i=1}^{n} F_{y,i}\right)^2\right]^{1/2}$$

The resultant direction with respect to the x-axis using four-quadrant angle functions is

$$\theta = \arctan\left(\sum_{i=1}^{n} F_{y,i} \Big/ \sum_{i=1}^{n} F_{x,i}\right)$$

The vector form of a force is

$$\boldsymbol{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

RESOLUTION OF A FORCE

$F_x = F\cos\theta_x$; $F_y = F\cos\theta_y$; $F_z = F\cos\theta_z$

$\cos\theta_x = F_x/F$; $\cos\theta_y = F_y/F$; $\cos\theta_z = F_z/F$

Separating a force into components (geometry of force is known $R = \sqrt{x^2 + y^2 + z^2}$)

$F_x = (x/R)F$; $F_y = (y/R)$F; $F_z = (z/R)F$

MOMENTS (COUPLES)

A system of two forces that are equal in magnitude, opposite in direction, and parallel to each other is called a *couple*.

A *moment* $\boldsymbol{M}$ is defined as the cross product of the *radius vector* $\boldsymbol{r}$ and the *force* $\boldsymbol{F}$ from a point to the line of action of the force.

$\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$; $M_x = yF_z - zF_y$,

$M_y = zF_x - xF_z$, and

$M_z = xF_y - yF_x$.

SYSTEMS OF FORCES

$$\boldsymbol{F} = \Sigma\,\boldsymbol{F}_n$$

$$\boldsymbol{M} = \Sigma\,(\boldsymbol{r}_n \times \boldsymbol{F}_n)$$

Equilibrium Requirements

$$\Sigma\,\boldsymbol{F}_n = 0$$

$$\Sigma\,\boldsymbol{M}_n = 0$$

CENTROIDS OF MASSES, AREAS, LENGTHS, AND VOLUMES

Formulas for centroids, moments of inertia, and first moment of areas are presented in the **MATHEMATICS** section for continuous functions. The following discrete formulas are for defined regular masses, areas, lengths, and volumes:

$\boldsymbol{r}_c = \Sigma\, m_n \boldsymbol{r}_n / \Sigma\, m_n$, where

m_n = the *mass of each particle* making up the system,

$\boldsymbol{r}_n$ = the *radius vector* to each particle from a selected reference point, and

$\boldsymbol{r}_c$ = the *radius vector* to the *center of the total mass* from the selected reference point.

The *moment of area* (M_a) is defined as

$$M_{ay} = \Sigma\, x_n a_n$$

$$M_{ax} = \Sigma\, y_n a_n$$

$$M_{az} = \Sigma\, z_n a_n$$

The *centroid of area* is defined as

$$\left.\begin{aligned} x_{ac} &= M_{ay}/A \\ y_{ac} &= M_{ax}/A \\ z_{ac} &= M_{az}/A \end{aligned}\right\} \text{with respect to center of the coordinate system}$$

where $A = \Sigma\, a_n$

The *centroid of a line* is defined as

$x_{lc} = (\Sigma\, x_n l_n)/L$, where $L = \Sigma\, l_n$

$y_{lc} = (\Sigma\, y_n l_n)/L$

$z_{lc} = (\Sigma\, z_n l_n)/L$

The *centroid of volume* is defined as

$x_{vc} = (\Sigma\, x_n v_n)/V$, where $V = \Sigma\, v_n$

$y_{vc} = (\Sigma\, y_n v_n)/V$

$z_{vc} = (\Sigma\, z_n v_n)/V$

MOMENT OF INERTIA

The *moment of inertia*, or the second moment of area, is defined as

$$I_y = \int x^2\, dA$$

$$I_x = \int y^2\, dA$$

The *polar moment of inertia* J of an area about a point is equal to the sum of the moments of inertia of the area about any two perpendicular axes in the area and passing through the same point.

$$I_z = J = I_y + I_x = \int (x^2 + y^2)\, dA$$

$$= r_p^{\,2} A, \text{ where}$$

r_p = the *radius of gyration* (see page 25).

Moment of Inertia Transfer Theorem

The moment of inertia of an area about any axis is defined as the moment of inertia of the area about a parallel centroidal axis plus a term equal to the area multiplied by the square of the perpendicular distance *d* from the centroidal axis to the axis in question.

$$I'_x = I_{x_c} + d_x^2 A$$

$$I'_y = I_{y_c} + d_y^2 A \text{, where}$$

d_x, d_y = distance between the two axes in question,

I_{x_c}, I_{y_c} = the moment of inertia about the centroidal axis, and

I_x', I_y' = the moment of inertia about the new axis.

Radius of Gyration

The *radius of gyration* r_p, r_x, r_y is the distance from a reference axis at which all of the area can be considered to be concentrated to produce the moment of inertia.

$$r_x = \sqrt{I_x/A}; \quad r_y = \sqrt{I_y/A}; \quad r_p = \sqrt{J/A}$$

Product of Inertia

The *product of inertia* (I_{xy}, etc.) is defined as:

$I_{xy} = \int xy dA$, with respect to the *xy*-coordinate system,

$I_{xz} = \int xz dA$, with respect to the *xz*-coordinate system, and

$I_{yz} = \int yz dA$, with respect to the *yz*-coordinate system.

The *transfer theorem* also applies:

$I'_{xy} = I_{x_c y_c} + d_x d_y A$ for the *xy*-coordinate system, etc.

where

d_x = *x*-axis distance between the two axes in question, and

d_y = *y*-axis distance between the two axes in question.

FRICTION

The largest frictional force is called the *limiting friction.* Any further increase in applied forces will cause motion.

$$F \le \mu N \text{, where}$$

F = friction force,

μ = *coefficient of static friction*, and

N = normal force between surfaces in contact.

SCREW THREAD (also see MECHANICAL ENGINEERING section)

For a *screw-jack, square thread,*

$$M = Pr \tan(\alpha \pm \phi) \text{, where}$$

+ is for screw tightening,

– is for screw loosening,

M = external moment applied to axis of screw,

P = load on jack applied along and on the line of the axis,

r = the mean thread radius,

α = the *pitch angle* of the thread, and

$\mu = \tan \phi$ = the appropriate coefficient of friction.

BELT FRICTION

$$F_1 = F_2 e^{\mu\theta} \text{, where}$$

F_1 = force being applied in the direction of impending motion,

F_2 = force applied to resist impending motion,

μ = coefficient of static friction, and

θ = the total *angle of contact* between the surfaces expressed in radians.

STATICALLY DETERMINATE TRUSS

Plane Truss

A plane truss is a rigid framework satisfying the following conditions:

1. The members of the truss lie in the same plane.
2. The members are connected at their ends by frictionless pins.
3. All of the external loads lie in the plane of the truss and are applied at the joints only.
4. The truss reactions and member forces can be determined using the equations of equilibrium.

   $\Sigma \mathbf{F} = 0$; $\Sigma \mathbf{M} = 0$
5. A truss is statically indeterminate if the reactions and member forces cannot be solved with the equations of equilibrium.

Plane Truss: Method of Joints

The method consists of solving for the forces in the members by writing the two equilibrium equations for each joint of the truss.

$$\Sigma F_V = 0 \text{ and } \Sigma F_H = 0 \text{, where}$$

F_H = horizontal forces and member components and

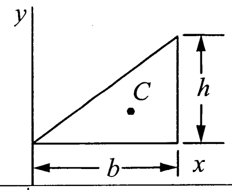
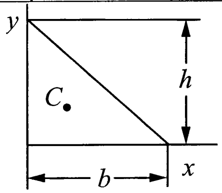
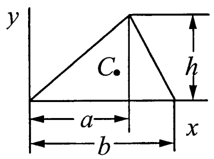
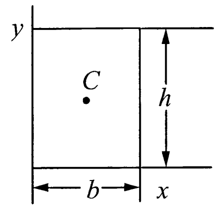
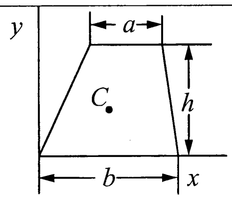
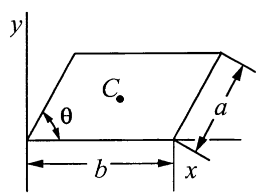
F_V = vertical forces and member components.

Plane Truss: Method of Sections

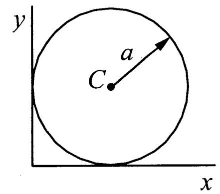
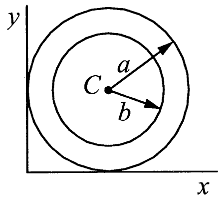
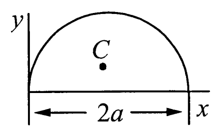
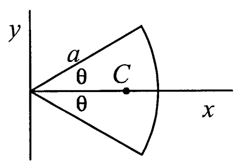
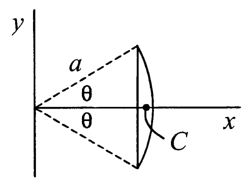
The method consists of drawing a free-body diagram of a portion of the truss in such a way that the unknown truss member force is exposed as an external force.

CONCURRENT FORCES

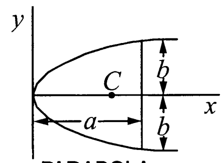
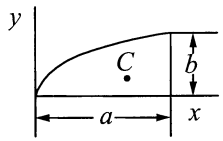
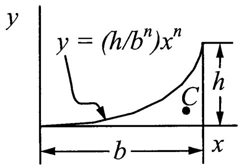
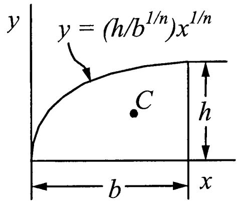
A concurrent-force system is one in which the lines of action of the applied forces all meet at one point. A *two-force* body in static equilibrium has two applied forces that are equal in magnitude, opposite in direction, and collinear. A *three-force* body in static equilibrium has three applied forces whose lines of action meet at a point. As a consequence, if the direction and magnitude of two of the three forces are known, the direction and magnitude of the third can be determined.

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = bh/2$ $x_c = 2b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/4$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/2$	$I_{x_cy_c} = Abh/36 = b^2h^2/72$ $I_{xy} = Abh/4 = b^2h^2/8$
	$A = bh/2$ $x_c = b/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = b^3h/36$ $I_x = bh^3/12$ $I_y = b^3h/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = b^2/18$ $r_x^2 = h^2/6$ $r_y^2 = b^2/6$	$I_{x_cy_c} = -Abh/36 = -b^2h^2/72$ $I_{xy} = Abh/12 = b^2h^2/24$
	$A = bh/2$ $x_c = (a+b)/3$ $y_c = h/3$	$I_{x_c} = bh^3/36$ $I_{y_c} = \left[bh\left(b^2 - ab + a^2\right)\right]/36$ $I_x = bh^3/12$ $I_y = \left[bh\left(b^2 + ab + a^2\right)\right]/12$	$r_{x_c}^2 = h^2/18$ $r_{y_c}^2 = \left(b^2 - ab + a^2\right)/18$ $r_x^2 = h^2/6$ $r_y^2 = \left(b^2 + ab + a^2\right)/6$	$I_{x_cy_c} = \left[Ah(2a-b)\right]/36$ $= \left[bh^2(2a-b)\right]/72$ $I_{xy} = \left[Ah(2a+b)\right]/12$ $= \left[bh^2(2a+b)\right]/24$
	$A = bh$ $x_c = b/2$ $y_c = h/2$	$I_{x_c} = bh^3/12$ $I_{y_c} = b^3h/12$ $I_x = bh^3/3$ $I_y = b^3h/3$ $J = \left[bh\left(b^2 + h^2\right)\right]/12$	$r_{x_c}^2 = h^2/12$ $r_{y_c}^2 = b^2/12$ $r_x^2 = h^2/3$ $r_y^2 = b^2/3$ $r_p^2 = \left(b^2 + h^2\right)/12$	$I_{x_cy_c} = 0$ $I_{xy} = Abh/4 = b^2h^2/4$
	$A = h(a+b)/2$ $y_c = \dfrac{h(2a+b)}{3(a+b)}$	$I_{x_c} = \dfrac{h^3\left(a^2 + 4ab + b^2\right)}{36(a+b)}$ $I_x = \dfrac{h^3(3a+b)}{12}$	$r_{x_c}^2 = \dfrac{h^2\left(a^2 + 4ab + b^2\right)}{18(a+b)}$ $r_x^2 = \dfrac{h^2(3a+b)}{6(a+b)}$	
	$A = ab \sin\theta$ $x_c = (b + a\cos\theta)/2$ $y_c = (a\sin\theta)/2$	$I_{x_c} = \left(a^3b\sin^3\theta\right)/12$ $I_{y_c} = \left[ab\sin\theta\left(b^2 + a^2\cos^2\theta\right)\right]/12$ $I_x = \left(a^3b\sin^3\theta\right)/3$ $I_y = \left[ab\sin\theta(b + a\cos\theta)^2\right]/3$ $-\left(a^2b^2\sin\theta\cos\theta\right)/6$	$r_{x_c}^2 = (a\sin\theta)^2/12$ $r_{y_c}^2 = \left(b^2 + a^2\cos^2\theta\right)/12$ $r_x^2 = (a\sin\theta)^2/3$ $r_y^2 = (b + a\cos\theta)^2/3$ $-(ab\cos\theta)/6$	$I_{x_cy_c} = \left(a^3b\sin^2\theta\cos\theta\right)/12$

Housner, George W. & Donald E. Hudson, *Applied Mechanics Dynamics*, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
	$A = \pi a^2$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi a^4/4$ $I_x = I_y = 5\pi a^4/4$ $J = \pi a^4/2$	$r_{x_c}^2 = r_{y_c}^2 = a^2/4$ $r_x^2 = r_y^2 = 5a^2/4$ $r_p^2 = a^2/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$
	$A = \pi(a^2 - b^2)$ $x_c = a$ $y_c = a$	$I_{x_c} = I_{y_c} = \pi\left(a^4 - b^4\right)/4$ $I_x = I_y = \frac{5\pi a^4}{4} - \pi a^2 b^2 - \frac{\pi b^4}{4}$ $J = \pi\left(a^4 - b^4\right)/2$	$r_{x_c}^2 = r_{y_c}^2 = \left(a^2 + b^2\right)/4$ $r_x^2 = r_y^2 = \left(5a^2 + b^2\right)/4$ $r_p^2 = \left(a^2 + b^2\right)/2$	$I_{x_c y_c} = 0$ $I_{xy} = Aa^2$ $= \pi a^2\left(a^2 - b^2\right)$
	$A = \pi a^2/2$ $x_c = a$ $y_c = 4a/(3\pi)$	$I_{x_c} = \frac{a^4\left(9\pi^2 - 64\right)}{72\pi}$ $I_{y_c} = \pi a^4/8$ $I_x = \pi a^4/8$ $I_y = 5\pi a^4/8$	$r_{x_c}^2 = \frac{a^2\left(9\pi^2 - 64\right)}{36\pi^2}$ $r_{y_c}^2 = a^2/4$ $r_x^2 = a^2/4$ $r_y^2 = 5a^2/4$	$I_{x_c y_c} = 0$ $I_{xy} = 2a^2/3$
CIRCULAR SECTOR	$A = a^2\theta$ $x_c = \frac{2a}{3}\frac{\sin\theta}{\theta}$ $y_c = 0$	$I_x = a^4(\theta - \sin\theta\cos\theta)/4$ $I_y = a^4(\theta + \sin\theta\cos\theta)/4$	$r_x^2 = \frac{a^2}{4}\frac{(\theta - \sin\theta\cos\theta)}{\theta}$ $r_y^2 = \frac{a^2}{4}\frac{(\theta + \sin\theta\cos\theta)}{\theta}$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
CIRCULAR SEGMENT	$A = a^2\left(\theta - \frac{\sin 2\theta}{2}\right)$ $x_c = \frac{2a}{3}\frac{\sin^3\theta}{\theta - \sin\theta\cos\theta}$ $y_c = 0$	$I_x = \frac{Aa^2}{4}\left[1 - \frac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $I_y = \frac{Aa^2}{4}\left[1 + \frac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$r_x^2 = \frac{a^2}{4}\left[1 - \frac{2\sin^3\theta\cos\theta}{3\theta - 3\sin\theta\cos\theta}\right]$ $r_y^2 = \frac{a^2}{4}\left[1 + \frac{2\sin^3\theta\cos\theta}{\theta - \sin\theta\cos\theta}\right]$	$I_{x_c y_c} = 0$ $I_{xy} = 0$

Housner, George W. & Donald E. Hudson, *Applied Mechanics Dynamics*, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.

Figure	Area & Centroid	Area Moment of Inertia	(Radius of Gyration)^2	Product of Inertia
 PARABOLA	$A = 4ab/3$ $x_c = 3a/5$ $y_c = 0$	$I_{x_c} = I_x = 4ab^3/15$ $I_{y_c} = 16a^3b/175$ $I_y = 4a^3b/7$	$r_{x_c}^2 = r_x^2 = b^2/5$ $r_{y_c}^2 = 12a^2/175$ $r_y^2 = 3a^2/7$	$I_{x_c y_c} = 0$ $I_{xy} = 0$
 HALF A PARABOLA	$A = 2ab/3$ $x_c = 3a/5$ $y_c = 3b/8$	$I_x = 2ab^3/15$ $I_y = 2ba^3/7$	$r_x^2 = b^2/5$ $r_y^2 = 3a^2/7$	$I_{xy} = Aab/4 = a^2b^2$
 n[th] DEGREE PARABOLA	$A = bh/(n+1)$ $x_c = \frac{n+1}{n+2}b$ $y_c = \frac{h}{2}\frac{n+1}{2n+1}$	$I_x = \frac{bh^3}{3(3n+1)}$ $I_y = \frac{hb^3}{n+3}$	$r_x^2 = \frac{h^2(n+1)}{3(3n+1)}$ $r_y^2 = \frac{n+1}{n+3}b^2$	
 n[th] DEGREE PARABOLA	$A = \frac{n}{n+1}bh$ $x_c = \frac{n+1}{2n+1}b$ $y_c = \frac{n+1}{2(n+2)}h$	$I_x = \frac{n}{3(n+3)}bh^3$ $I_y = \frac{n}{3n+1}b^3h$	$r_x^2 = \frac{n+1}{3(n+1)}h^2$ $r_y^2 = \frac{n+1}{3n+1}b^2$	

Housner, George W. & Donald E. Hudson, *Applied Mechanics Dynamics*, D. Van Nostrand Company, Inc., Princeton, NJ, 1959. Table reprinted by permission of G.W. Housner & D.E. Hudson.