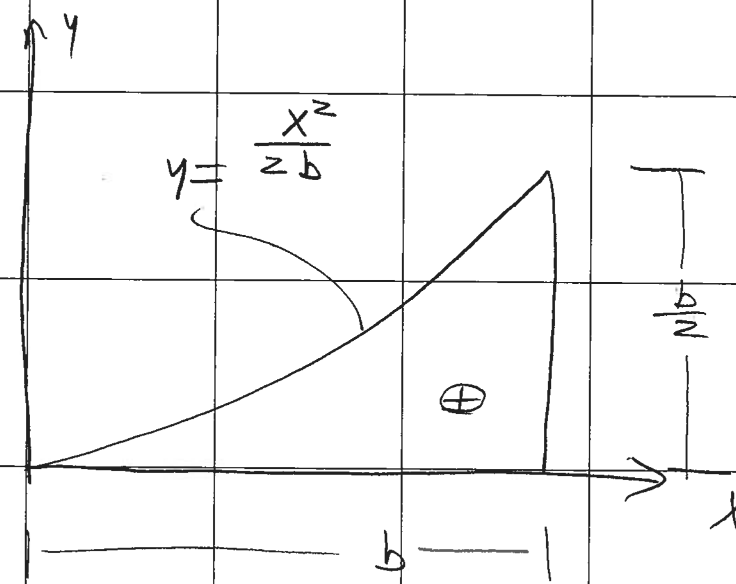
Calculate the radius of gyration for the shaded area shown with respect to an axis that passes through the centroid of the area and normal to the plane of the area.

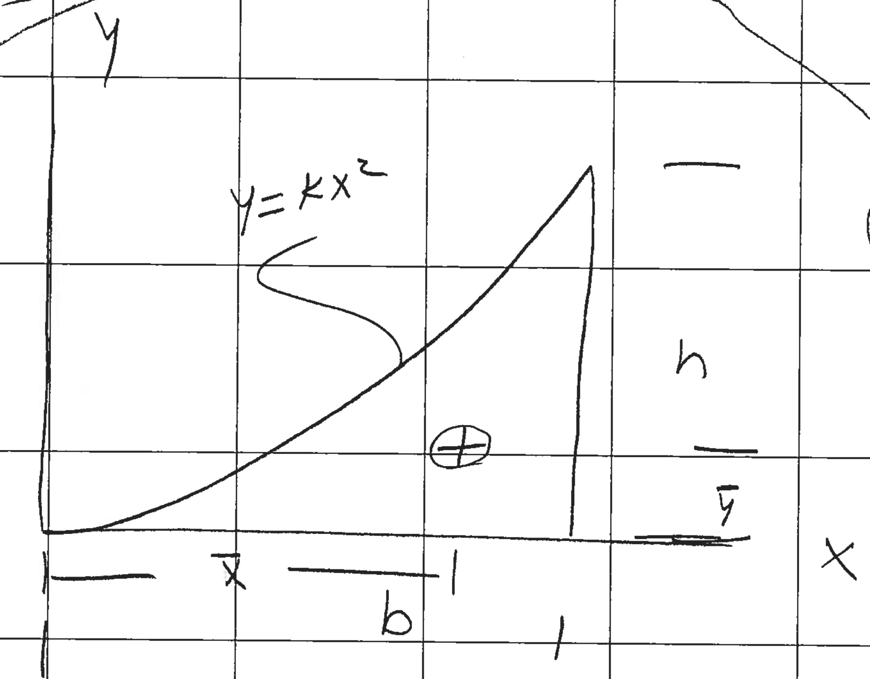


The axis in question is the centroidal Z-axis. The easiest way to calculate this value is with the following relationship

(1) $$I_{\bar{z}} = I_{\bar{x}} + I_{\bar{y}}$$

Because the shaded area is a parabolic spandrel of the form $y = k x^2$, then formulas developed for Parabolic spandrels will be used

These results are summarized as follows



$$A = \frac{bh}{3}, \quad \bar{x} = \frac{3}{4} b, \quad \bar{y} = \frac{3}{10} h$$

$$I_x = \frac{b h^3}{21}, \quad I_y = \frac{h b^3}{5}$$

Calculated in Previous examples

In the current case, $b = b$ and $h = \frac{b}{2}$

Therefore

(2) $$A = \frac{bh}{3} = \frac{b \frac{b}{2}}{3} = \frac{b^2}{6}$$

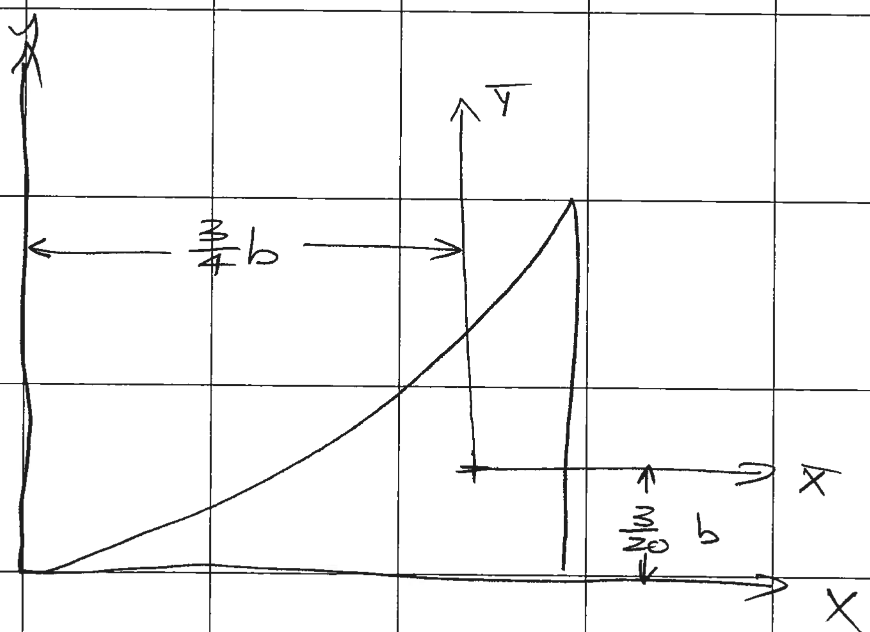
(3) $\bar{x} = \frac{3}{4} b$

(4) $\bar{y} = \frac{3}{10} h = \frac{3}{10} \frac{b}{2} = \frac{3}{20} b$

(5) $I_x = \frac{b h^3}{21} = \frac{b \left(\frac{b}{2}\right)^3}{21} = \frac{b^4}{168}$

(6) $I_y = \frac{h b^3}{5} = \frac{\frac{b}{2} b^3}{5} = \frac{b^4}{10}$

Then $I_{\bar{x}}$ and $I_{\bar{y}}$ are calculated with the parallel axis theorem as follows.



$I_{\bar{x}}$

(7) $$I_x = I_{\bar{x}} + A d^2$$

(8) $$\frac{b^4}{168} = I_{\bar{x}} + \frac{b^2}{6}\left(\frac{3}{20} b\right)^2$$

(9) $$I_{\bar{x}} = \frac{b^4}{168} - \frac{9 b^4}{2400}$$

(10) $$I_{\bar{x}} = \frac{b^4}{16,800}\left[100 - 63\right]$$

(11) $$I_{\bar{x}} = \frac{37 b^4}{16,800}$$

$I_{\bar{y}}$

(12) $$I_y = I_{\bar{y}} + A d^2$$

(13) $$\frac{b^4}{10} = I_{\bar{y}} + \frac{b^2}{6}\left(\frac{3}{4} b\right)^2$$

(14) $$I_{\bar{y}} = \frac{b^4}{10} - \frac{9 b^4}{96}$$

(15) $$I_{\bar{y}} = \frac{b^4}{960}\left[96 - 90\right]$$

(16) $$I_{\bar{y}} = \frac{6 b^4}{960}$$

(17) $$I_{\bar{y}} = \frac{b^4}{160}$$

I_z

(18) $I_{\bar{z}} = I_{\bar{x}} + I_{\bar{y}}$

(19) $I_{\bar{z}} = \frac{37b^4}{16,800} + \frac{b^4}{160}$

(20) $I_z = \frac{b^4}{16,800}\left[37 + 105\right]$

(21) $I_{\bar{z}} = \frac{142}{16,800} b^4$

(22) $I_{\bar{z}} = \frac{71}{8,400} b^4$

Finally calculate the corresponding radius of gyration for $I_{\bar{z}}$ as follows

(23) $k_z = \sqrt{\frac{I_{\bar{z}}}{A}}$

(24) $k_z = \sqrt{\frac{\frac{71 b^4}{8400}}{\frac{b^2}{6}}}$

(25) $k_z = \sqrt{\frac{71 b^4\, 6}{8400\, b^2}}$

(26) $k_z = \sqrt{\frac{426}{8400} b^2}$

(27) $K_z = .2252\,b$