10-1* Determine the second moment of area for the isosceles triangle shown in Fig. P10-1 with respect to
(a) The base of the triangle (the x-axis).
(b) An axis through the centroid parallel to the base.

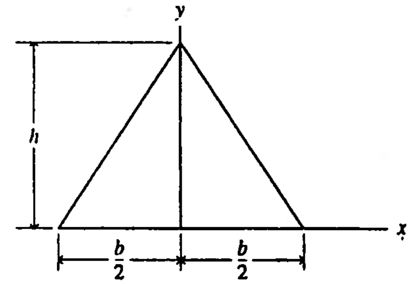
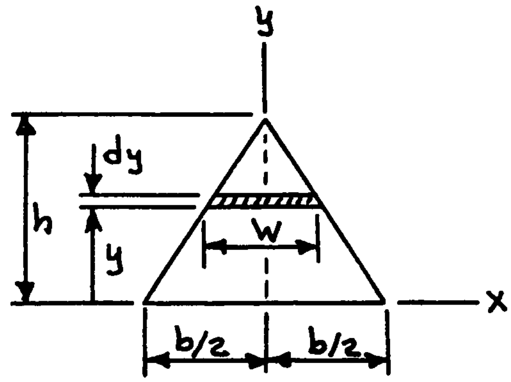


Fig. P10-1

SOLUTION



From similar triangles:

$$\frac{w}{h - y} = \frac{b}{h} \qquad w = \frac{b}{h}(h - y)$$

(a) $$I_x = \int_A y^2\, dA = \int_A y^2\, (w\, dy)$$

$$= \int_0^h \frac{b}{h}(h - y)\, y^2\, dy$$

$$= \frac{b}{h}\int_0^h (hy^2 - y^3)\, dy$$

$$= \frac{b}{h}\left[\frac{hy^3}{3} - \frac{y^4}{4}\right]_0^h = \frac{b}{h}\left(\frac{h^4}{12}\right) = \frac{bh^3}{12}$$ Ans.

(b) $$I_{xC} = I_x - d_x^2 A = \frac{bh^3}{12} - \left(\frac{h}{3}\right)^2\left(\frac{bh}{2}\right) = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{bh^3}{36}$$ Ans.

10-5* Determine the second moment of area for the shaded region shown in Fig. P10-5 with respect to
(a) The x-axis.
(b) The y-axis.

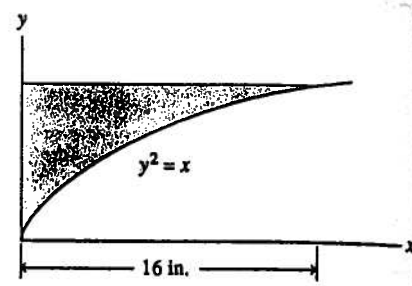
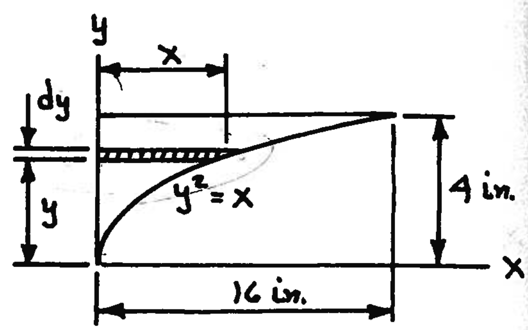


Fig. P10-5

SOLUTION



(a) From the curve:

$$x = y^2$$

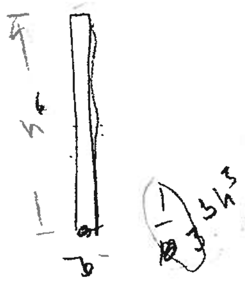
$$dA = x\,dy = y^2\,dy$$

$$I_x = \int_A y^2\,dA$$

$$= \int_0^4 y^4\,dy = \left[\frac{y^5}{5}\right]_0^4 = 204.8 \text{ in.}^4 \cong 205 \text{ in.}^4 \qquad \text{Ans.}$$

$I_y = \int x^2 dA$

(b) From the results of Example Problem 10-1:



$$dI_y = \frac{1}{3}bh^3 = \frac{1}{3}x^3\,dy = \frac{1}{3}y^6\,dy$$

$$I_y = \int_A dI_y$$

$dI_y = \frac{1}{3} dy\, x^3$

$$= \int_0^4 \frac{1}{3}y^6\,dy = \left[\frac{y^7}{21}\right]_0^4 = 780.2 \text{ in.}^4 \cong 780 \text{ in.}^4 \qquad \text{Ans.}$$

$I_y = \int \frac{1}{3} dy\, x^3 = \int_0^4 \frac{1}{3} y^6\, dy$ $\qquad \int \frac{1}{4} y^6\, dy$

10-7 Determine the second moment of area for the shaded region shown in Fig. P10-7 with respect to
(a) The x-axis.
(b) The y-axis.

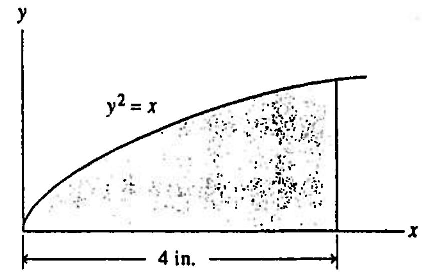
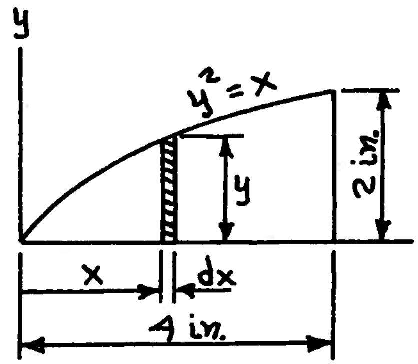


Fig. P10-7

SOLUTION



(a) From the results of Example Problem 10-1:

$$dI_x = \frac{1}{3}bh^3 = \frac{1}{3}y^3\,dx = \frac{1}{3}x^{3/2}\,dx$$

$$I_x = \int_A dI_x = \int_0^4 \frac{1}{3}x^{3/2}\,dx$$

$$= \frac{1}{3}\left[\frac{2}{5}x^{5/2}\right]_0^4 = 4.267\text{ in.}^4 \cong 4.27\text{ in.}^4$$ Ans.

(b) From the curve:

$$y^2 = x$$

$$dA = y\,dx = x^{1/2}\,dx$$

$$I_y = \int_A x^2\,dA = \int_0^4 x^2\left(x^{1/2}\,dx\right)$$

$$= \int_0^4 x^{5/2}\,dx$$

$$= \frac{2}{7}\left[x^{7/2}\right]_0^4 = 36.57\text{ in.}^4 \cong 36.6\text{ in.}^4$$ Ans.

10-8 Determine the second moment of area for the shaded region shown in Fig. P10-8 with respect to
(a) The x-axis.
(b) The y-axis.

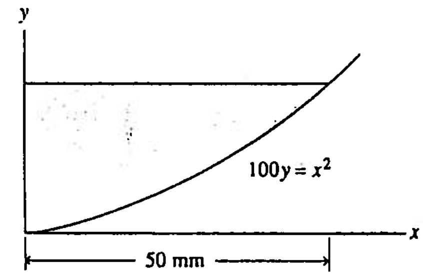
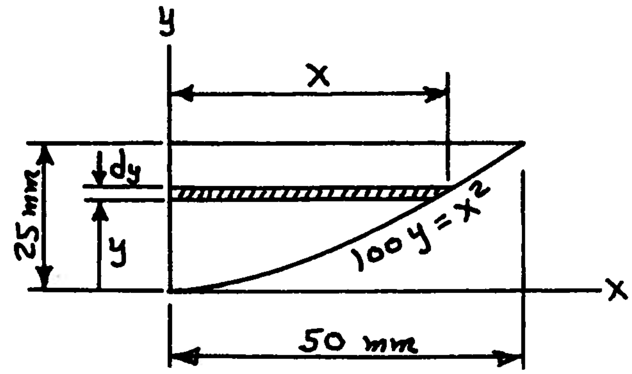


Fig. P10-8

SOLUTION



(a) From the curve:

$$100y = x^2$$

$$dA = x\ dy = 10y^{1/2}\ dy$$

$$I_x = \int_A y^2\ dA = \int_0^{25} y^2\left(10y^{1/2}\ dy\right)$$

$$= \int_0^{25} 10y^{5/2}\ dy$$

$$= 10\left[\frac{2}{7}\ y^{7/2}\right]_0^{25} = 0.223(10^6)\ \text{mm}^4 \qquad \text{Ans.}$$

(b) From the results of Example Problem 10-1:

$$dI_y = \frac{1}{3}\ bh^3 = \frac{1}{3}\ x^3\ dy = \frac{1}{3}(1000\ y^{3/2})\ dy$$

$$I_y = \int_A dI_y = \int_0^{25} \frac{1}{3}\ (1000y^{3/2})\ dy$$

$$= \frac{1000}{3}\left[\frac{2}{5}\ y^{5/2}\right]_0^{25} = 0.417(10^6)\ \text{mm}^4 \qquad \text{Ans.}$$

10-17* Determine the radii of gyration for the rectangular area shown in Fig. P10-17 with respect to
(a) The x- and y-axes shown on the figure.
(b) Horizontal and vertical centroidal axes.

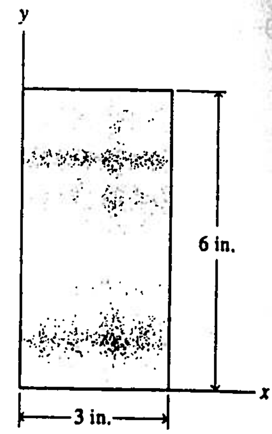
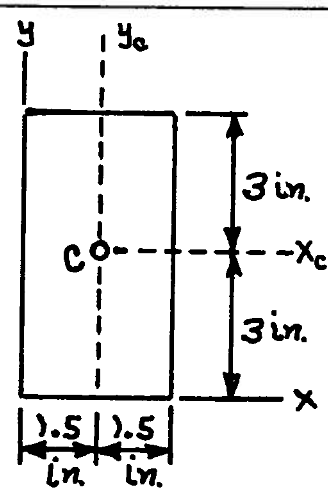


Fig. P10-17

SOLUTION



$$A = bh = 3(6) = 18 \text{ in.}^2$$

(a) From the results of Example Problem 10-1:

$$I_x = \frac{1}{3}bh^3 = \frac{1}{3}(3)(6)^3 = 216 \text{ in.}^4$$

$$I_y = \frac{1}{3}hb^3 = \frac{1}{3}(6)(3)^3 = 54 \text{ in.}^4$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{216}{18}} = 3.46 \text{ in.}$$ Ans.

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{54}{18}} = 1.732 \text{ in.}$$ Ans.

(b) From the results of Example Problem 10-1:

$$I_{xC} = \frac{1}{12}bh^3 = \frac{1}{12}(3)(6)^3 = 54 \text{ in.}^4$$

$$I_{yC} = \frac{1}{12}hb^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in.}^4$$

$$k_{xC} = \sqrt{\frac{I_{xC}}{A}} = \sqrt{\frac{54}{18}} = 1.732 \text{ in.}$$ Ans.

$$k_{yC} = \sqrt{\frac{I_{yC}}{A}} = \sqrt{\frac{13.5}{18}} = 0.866 \text{ in.}$$ Ans.

10-26 Determine the radius of gyration for the shaded area shown in Fig. P10-26 with respect to an axis through the centroid of the area and normal to the plane of the area.

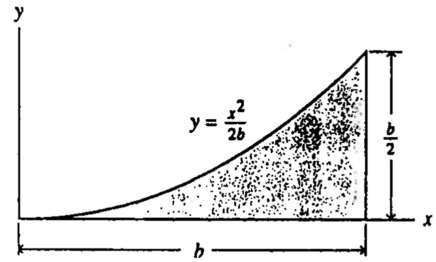
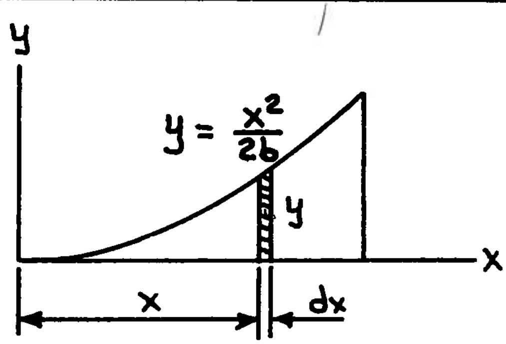


Fig. P10-26

SOLUTION



$$A = \int_A y\,dx = \int_0^b \frac{x^2}{2b}\,dx = \left[\frac{x^3}{6b}\right]_0^b = \frac{b^2}{6}$$

$$Ax_C = \int_A x\,dA = \int_0^b \frac{x^3}{2b}\,dx = \left[\frac{x^4}{8b}\right]_0^b = \frac{b^3}{8} \qquad x_C = \frac{b^3/8}{b^2/6} = \frac{3b}{4}$$

$$Ay_C = \int_A \frac{y}{2}\,dA = \int_0^b \frac{x^4}{8b^2}\,dx = \left[\frac{x^5}{40b^2}\right]_0^b = \frac{b^3}{40} \qquad y_C = \frac{b^3/40}{b^2/6} = \frac{3b}{20}$$

$$dI_x = \frac{1}{3}\left(\frac{x^2}{2b}\right)^3 dx = \frac{x^6}{24b^3}\,dx$$

$$I_x = \int_A dI_x = \int_0^b \frac{x^6}{24b^3}\,dx = \left[\frac{x^7}{168b^3}\right]_0^b = \frac{b^4}{168}$$

$$I_y = \int_A x^2\,dA = \int_0^b x^2\left(\frac{x^2}{2b}\right)dx = \int_0^b \frac{x^4}{2b}\,dx = \left[\frac{x^5}{10b}\right]_0^b = \frac{b^4}{10}$$

$$I_{xC} = I_x - d_x^2 A = \frac{b^4}{168} - \left(\frac{3b}{20}\right)^2\left(\frac{b^2}{6}\right) = \frac{37b^4}{16{,}800}$$

$$I_{yC} = I_y - d_y^2 A = \frac{b^4}{10} - \left(\frac{3b}{4}\right)^2\left(\frac{b^2}{6}\right) = \frac{b^4}{160}$$

$$J_{zC} = I_{xC} + I_{yC} = \frac{37b^4}{16800} + \frac{b^4}{160} = \frac{71b^4}{8400}$$

$$k_{zC} = \sqrt{\frac{J_{zC}}{A}} = \sqrt{\frac{71b^4/8400}{b^2/6}} = \sqrt{\frac{71b^2}{1400}} = 0.2252b \cong 0.225b$$ Ans.

10-27* Determine the polar radius of gyration for the shaded area shown in Fig. P10-27 with respect to an axis through the origin of the xy-coordinate system and normal to the plane of the area.

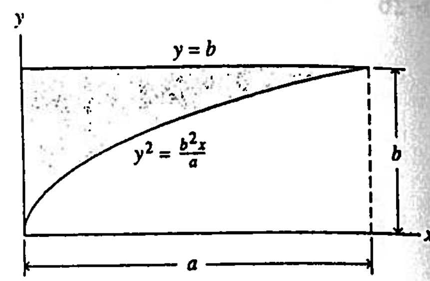
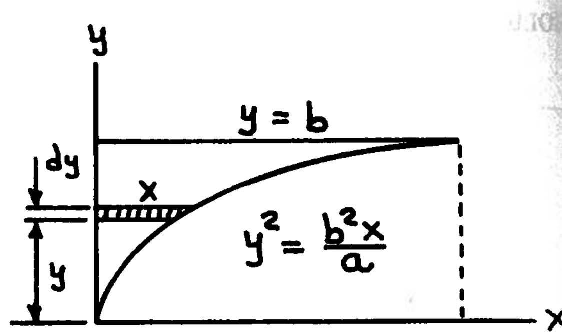


Fig. P10-27

SOLUTION



$$x = \frac{ay^2}{b^2}$$

$$dA = x\,dy = \frac{ay^2}{b^2}\,dy$$

$$A = \int_0^b \frac{ay^2}{b^2}\,dy = \frac{a}{b^2}\left[\frac{y^3}{3}\right]_0^b = \frac{1}{3}\,ab$$

$$I_x = \int_A y^2\,dA = \int_0^b y^2\left[\frac{ay^2}{b^2}\right]dy = \frac{a}{b^2}\left[\frac{y^5}{5}\right]_0^b = \frac{1}{5}\,ab^3$$

$$I_y = \int_A \frac{1}{3}\,x^3\,dy = \frac{1}{3}\left[\frac{ay^2}{b^2}\right]dy = \frac{a^3}{3b^6}\left[\frac{y^7}{7}\right]_0^b = \frac{1}{21}\,a^3b$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{ab^3/5}{ab/3}} = \frac{\sqrt{15}}{5}\,b$$

$$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{a^3b/21}{ab/3}} = \frac{\sqrt{7}}{7}\,a$$

$$k_z^2 = k_x^2 + k_y^2 = \frac{15b^2}{25} + \frac{7a^2}{49} = \frac{21b^2 + 5a^2}{35}$$

$$k_z = \sqrt{\frac{21b^2 + 5a^2}{35}}$$ Ans.

10-33* Determine the second moments with respect to x (horizontal) and y (vertical) axes through the centroid of the shaded area shown in Fig. P10-33.

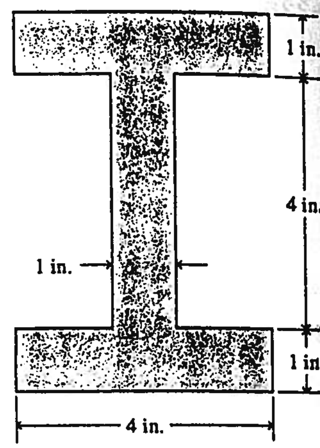
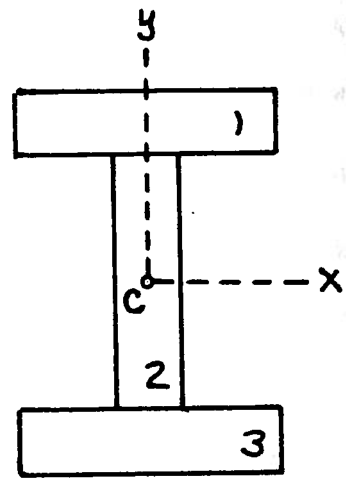


Fig. P10-33

SOLUTION



$$I_{xC1} = \frac{1}{12}(4)(1)^3 + (2.5)^2(4) = 25.33 \text{ in.}^4$$

$$I_{xC2} = \frac{1}{12}(1)(4)^3 = 5.33 \text{ in.}^4$$

$$I_{xC3} = \frac{1}{12}(4)(1)^3 + (-2.5)^2(4) = 25.33 \text{ in.}^4$$

$$I_{xC} = I_{xC1} + I_{xC2} + I_{xC3}$$

$$= 25.33 + 5.33 + 25.33 = 56.0 \text{ in.}^4$$ Ans.

$$I_{yC1} = \frac{1}{12}(1)(4)^3 = 5.333 \text{ in.}^4$$

$$I_{yC2} = \frac{1}{12}(4)(1)^3 = 0.333 \text{ in.}^4$$

$$I_{yC3} = \frac{1}{12}(1)(4)^3 = 5.333 \text{ in.}^4$$

$$I_{yC} = I_{yC1} + I_{yC2} + I_{yC3}$$

$$= 5.333 + 0.333 + 5.333 = 11.00 \text{ in.}^4$$ Ans.

10-34* Determine the second moments with respect to x (horizontal) and y (vertical) axes through the centroid of the shaded area shown in Fig. P10-34.

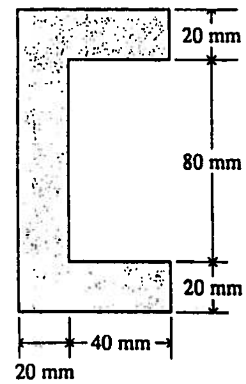
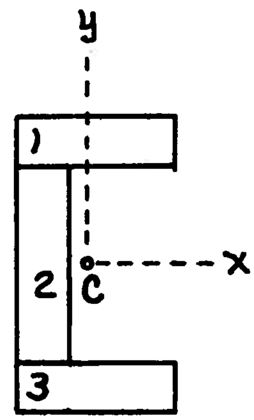


Fig. P10-34

SOLUTION



$$I_{xC1} = \frac{1}{12}(60)(20)^3 + (50)^2(1200) = 3.040(10^6)\ \text{mm}^4$$

$$I_{xC2} = \frac{1}{12}(20)(80)^3 = 0.853(10^6)\ \text{mm}^4$$

$$I_{xC3} = \frac{1}{12}(60)(20)^3 + (-50)^2(1200) = 3.040(10^6)\ \text{mm}^4$$

$$I_{xC} = I_{xC1} + I_{xC2} + I_{xC3} = 3.040(10^6) + 0.853(10^6) + 3.040(10^6)$$

$$= 6.933(10^6)\ \text{mm}^4 \cong 6.933(10^6)\ \text{mm}^4$$ Ans.

$$A = 20(60) + 20(80) + 20(60) = 4000\ \text{mm}^2$$

$$Ax_C = 20(60)(30) + 20(80)(10) + 20(60)(30) = 88{,}000\ \text{mm}^3$$

$$x_C = \frac{88{,}000}{4000} = 22.0\ \text{mm} \qquad y_C = 60\ \text{mm (By Symmetry)}$$

$$I_{yC1} = \frac{1}{12}(60)^3(20) + (8)^2(1200) = 0.4368(10^6)\ \text{mm}^4$$

$$I_{yC2} = \frac{1}{12}(20)^3(80) + (-12)^2(1600) = 0.2837(10^6)\ \text{mm}^4$$

$$I_{yC3} = \frac{1}{12}(60)^3(20) + (8)^2(1200) = 0.4368(10^6)\ \text{mm}^4$$

$$I_{yC} = I_{yC1} + I_{yC2} + I_{yC3} = 0.4368(10^6) + 0.2837(10^6) + 0.4368(10^6)$$

$$= 1.1573(10^6)\ \text{mm}^4 \cong 1.157(10^6)\ \text{mm}^4$$ Ans.

10-37* Determine the second moments with respect to x (horizontal) and y (vertical) axes through the centroid of the two 10 x 1-in. steel plates that are welded to the flanges of an S18 x 70 I-beam as shown in Fig. P10-37.

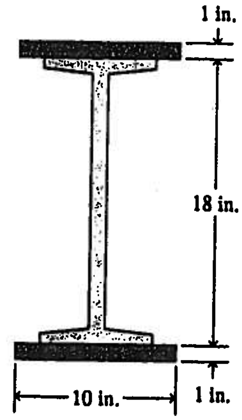


Fig. P10-37

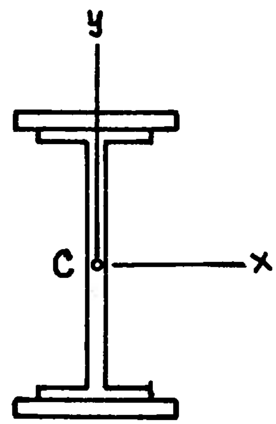
SOLUTION

From Table 10-2A:

For an S18 × 70 beam

$I_x = 926 \text{ in.}^4$

$I_y = 24.1 \text{ in.}^4$



$$I_{xC} = 926 + 2\left[\frac{10(1)^3}{12} + (9.5)^2(10)\right]$$

$$= 2733 \text{ in.}^4 \cong 2730 \text{ in.}^4$$ Ans.

$$I_{yC} = 24.1 + 2\left[\frac{1(10)^3}{12}\right] = 190.8 \text{ in.}^4$$ Ans.

10-38* Determine the second moments with respect to x (horizontal) and y (vertical) axes through the centroid of the two 250 x 25-mm steel plates and two C254 x 45 channels that are welded together to form the cross section shown in Fig. P10-38.

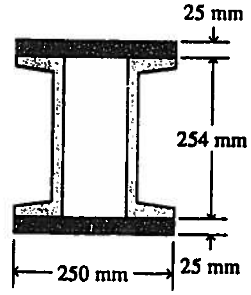
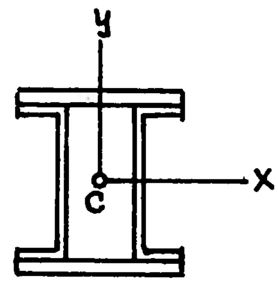


Fig. P10-38

SOLUTION



From Table 10-2B:

For a C254 × 45 channel:

$A = 5690 \text{ mm}^2$

$I_x = 42.9(10^6) \text{ mm}^4$

$I_y = 1.64(10^6) \text{ mm}^4$

$$I_{xC} = 2(42.9)(10^6) + 2\left[\frac{250(25)^3}{12} + (139.5)^2(25)(250)\right]$$

$$= 329.7(10^6) \text{ mm}^4 \cong 330(10^6) \text{ mm}^4 \quad \text{Ans.}$$

$$I_{yC} = 2\left[1.64(10^6) + (64.5)^2(5690)\right] + 2\left[\frac{(250)^3}{12}\right]$$

$$= 115.73(10^6) \text{ mm}^4 \cong 115.7(10^6) \text{ mm}^4 \quad \text{Ans.}$$

10-47 Determine the second moments with respect to the x-and y-axes shown on the figure for the shaded area shown in Fig. P10-47.

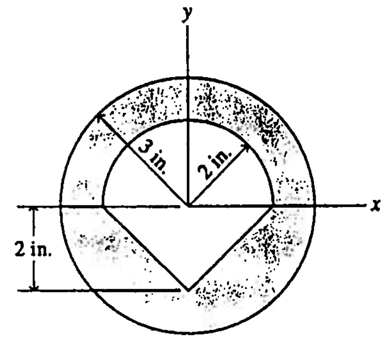


Fig. P10-47

SOLUTION

$$I_x = \frac{\pi(3)^4}{4} - \frac{\pi(2)^4}{8} - \frac{4(2)^3}{12} = 54.67 \text{ in.}^4 \cong 54.7 \text{ in.}^4$$ Ans.

$$I_y = \frac{\pi(3)^4}{4} - \frac{\pi(2)^4}{8} - \left[\frac{2(2)^3}{12} + \frac{2(2)^3}{12}\right] = 54.67 \text{ in.}^4 \cong 54.7 \text{ in.}^4$$ Ans.

10-48 Determine the second moments with respect to the x-and y-axes shown on the figure for the shaded area shown in Fig. P10-48.

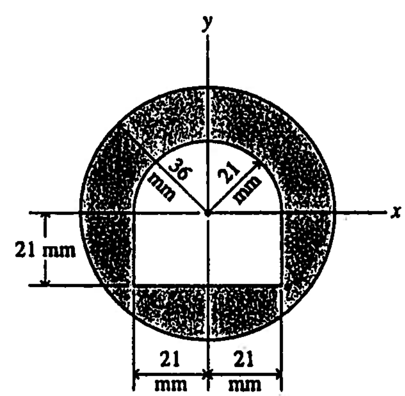


Fig. P10-48

SOLUTION

$$I_x = \frac{\pi(36)^4}{4} - \frac{\pi(21)^4}{8} - \frac{42(21)^3}{12}$$

$$= 1.1131(10^6) \text{ mm}^4 \cong 1.113(10^6) \text{ mm}^4$$ Ans.

$$I_y = \frac{\pi(36)^4}{4} - \frac{\pi(21)^4}{8} - \frac{21(42)^3}{12}$$

$$= 1.1131(10^6) \text{ mm}^4 \cong 1.113(10^6) \text{ mm}^4$$ Ans.

10-51 For the shaded area shown in Fig. P10-51, determine the second moments with respect to
(a) The x- and y-axes shown on the figure.
(b) The x-and y-axes through the centroid of the area.

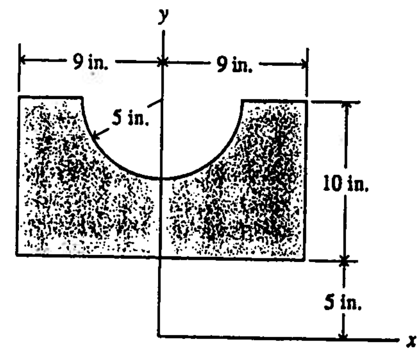


Fig. P10-51

SOLUTION

For The rectangle:

$$A = 18(10) = 180 \text{ in}^2$$

$$I_{xC} = \frac{1}{12}(18)(10)^3 = 1500 \text{ in}^4$$

$$I_{yC} = \frac{1}{12}(10)(18)^3 = 4860 \text{ in}^4$$

$$d_x = 10 \text{ in}$$

For the semicircle:

$$A = \frac{\pi(5)^2}{2} = 39.270 \text{ in}^2$$

$$I_{xC} = \frac{\pi(5)^2}{8} - \frac{8(5)^4}{9\pi} = 68.60 \text{ in}^4$$

$$I_{yC} = \frac{\pi(5)^4}{8} = 245.44 \text{in}^4$$

$$d_x = 15 - \frac{4(5)}{3\pi} = 12.878 \text{ in.}$$

(a) $I_x = 1500 + (10)^2(180) - 68.60 - (12.878)^2(39.270)$

$= 12{,}919 \text{ in}^4 \cong 12{,}920 \text{ in}^4$ Ans.

$I_y = 4860 - 245.44 = 4614.6 \text{ in}^4 \cong 4610 \text{ in}^4$ Ans.

(b) $A = 180 - 39.270 = 140.73 \text{ in}^2$

$Ay_C = 180(10) - 39.27(12.878) = 1294.28 \text{ in}^3$

$$y_C = \frac{1294.28}{140.73} = 9.197 \text{ in.}$$

(b) $I_{xC} = I_x - d_x^2 A = 12{,}920 - (9.197)^2(140.73)$

$= 1016.4 \text{ in}^4 \cong 1016 \text{ in}^4$ Ans.

$I_{yC} = I_y = 4610 \text{ in}^4$ Ans.

10-52 For the shaded area shown in Fig. P10-52, determine the second moments with respect to
(a) The x- and y-axes shown on the figure.
(b) The x-and y-axes through the centroid of the area.

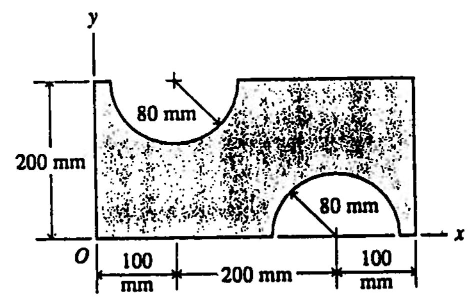


Fig. P10-52

SOLUTION

For The rectangle:

$$A = 200(400) = 80{,}000 \text{ mm}^2 \qquad I_x = \frac{1}{3}(400)(200)^3 = 1066.67(10^6) \text{ mm}^4$$

$$I_y = \frac{1}{3}(200)(400)^3 = 4266.67(10^6) \text{ mm}^4$$

For the semicircles:

$$A = \frac{\pi(80)^2}{2} = 10{,}053 \text{ mm}^2$$

$$I_{xC} = \frac{\pi(80)^4}{8} - \frac{8(80)^4}{9\pi} = 4.496(10^6) \text{ mm}^4 \qquad d_{x2} = 200 - \frac{4(80)}{3\pi} = 166.05 \text{ mm}$$

$$I_{yC} = \frac{\pi(80)^4}{8} = 16.085(10^6) \text{ mm}^4 \qquad d_{x3} = \frac{4(80)}{3\pi} = 33.95 \text{ mm}$$

(a) $I_x = 1066.67(10^6) - 4.496(10^6) - (166.05)^2(10{,}053)$

$- 4.496(10^6) - (33.95)^2(10{,}053)$

$= 768.90(10^6) \text{ mm}^4 \cong 769(10^6) \text{ mm}^4$ Ans.

$I_y = 4266.67(10^6) - 16.085(10^6) - (100)^2(10{,}053)$

$- 16.085(10^6) - (300)^2(10{,}053)$

$= 3229.2(10^6) \text{ mm}^4 \cong 3230(10^6) \text{ mm}^4$ Ans.

(b) $A = 80{,}000 - 2(10{,}053) = 59{,}984 \text{ mm}^2$

$I_{xC} = 768.90(10^6) - (100)^2(59{,}894)$

$= 169.96(10^6) \text{ mm}^4 \cong 170.0(10^6) \text{ mm}^4$ Ans.

$I_{yC} = 3229.2(10^6) - (200)^2(59{,}894)$

$= 833.44(10^6) \text{ mm}^4 \cong 833(10^6) \text{ mm}^4$ Ans.