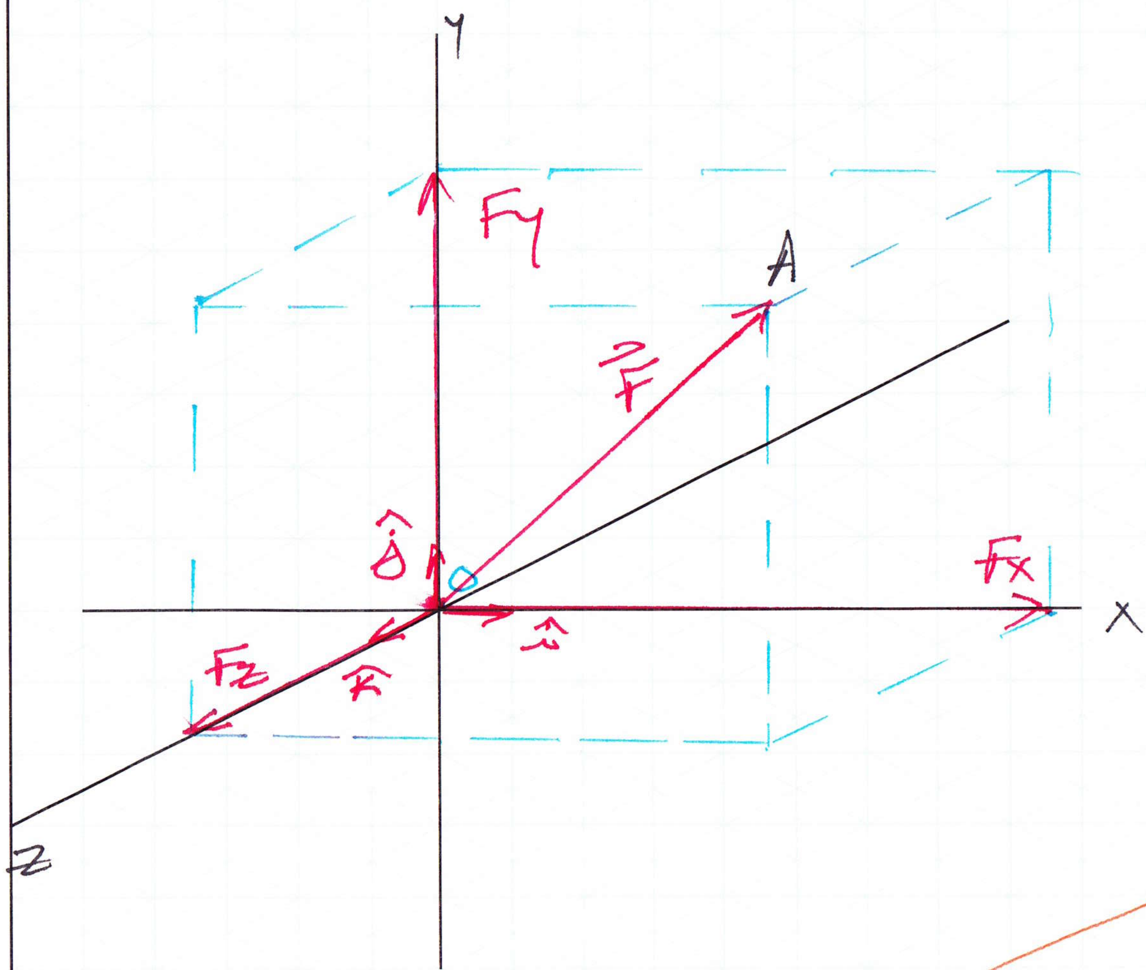
Rectangular Components of a Force in Space



3-D

$\vec{F}$ is directed along $\overrightarrow{OA}$

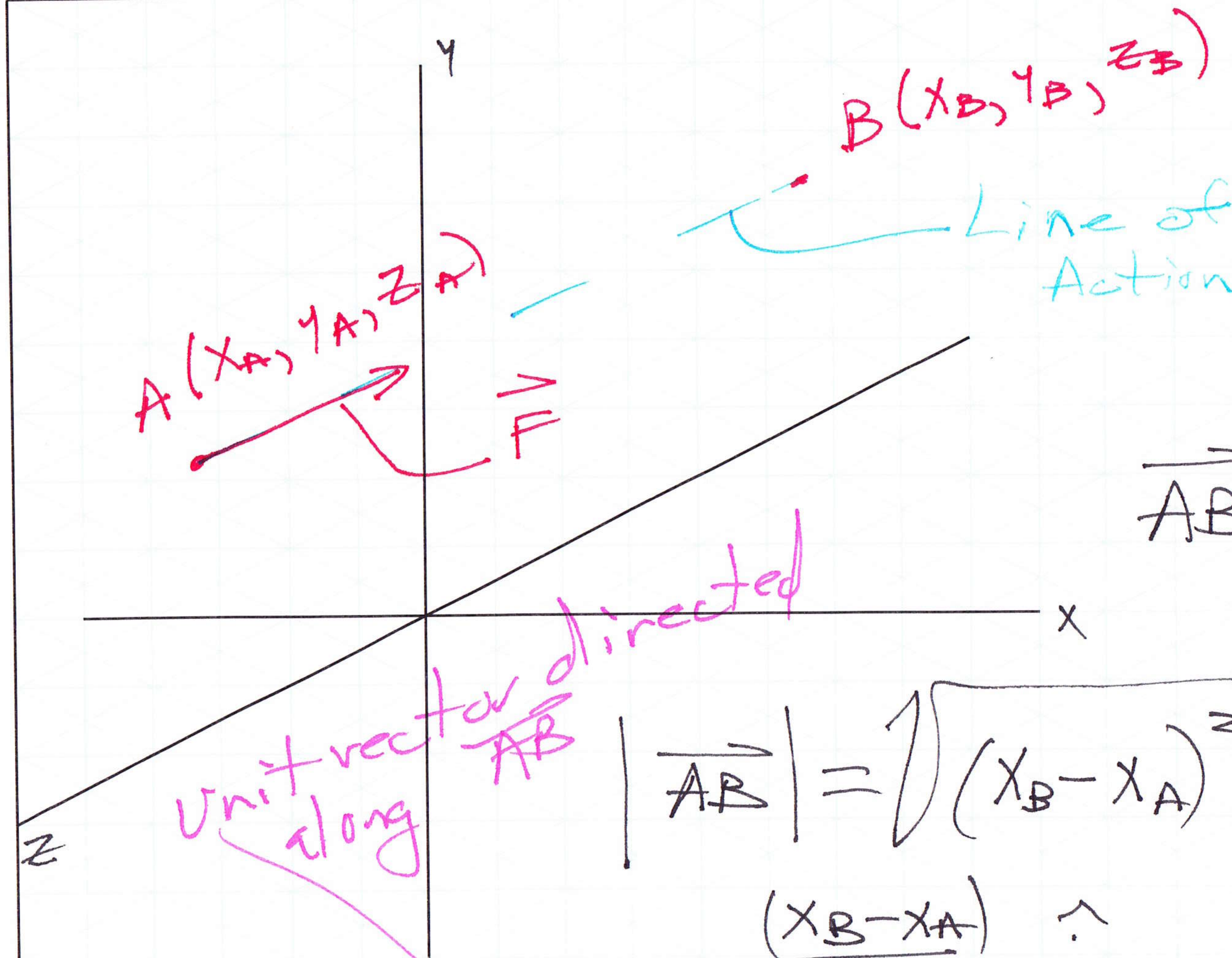
unit vectors

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

scalar components

Vector component in the x direction

- Point to Point
- Direction Cosines θ_x, θ_y, and θ_z
- Azimuth-Elevation
- Slope



Point to Point

A Force of magnitude F acting from A to B

$$\overrightarrow{AB} = (x_B - x_A)\,\hat{i} + (y_B - y_A)\,\hat{j} + (z_B - z_A)\,\hat{k}$$

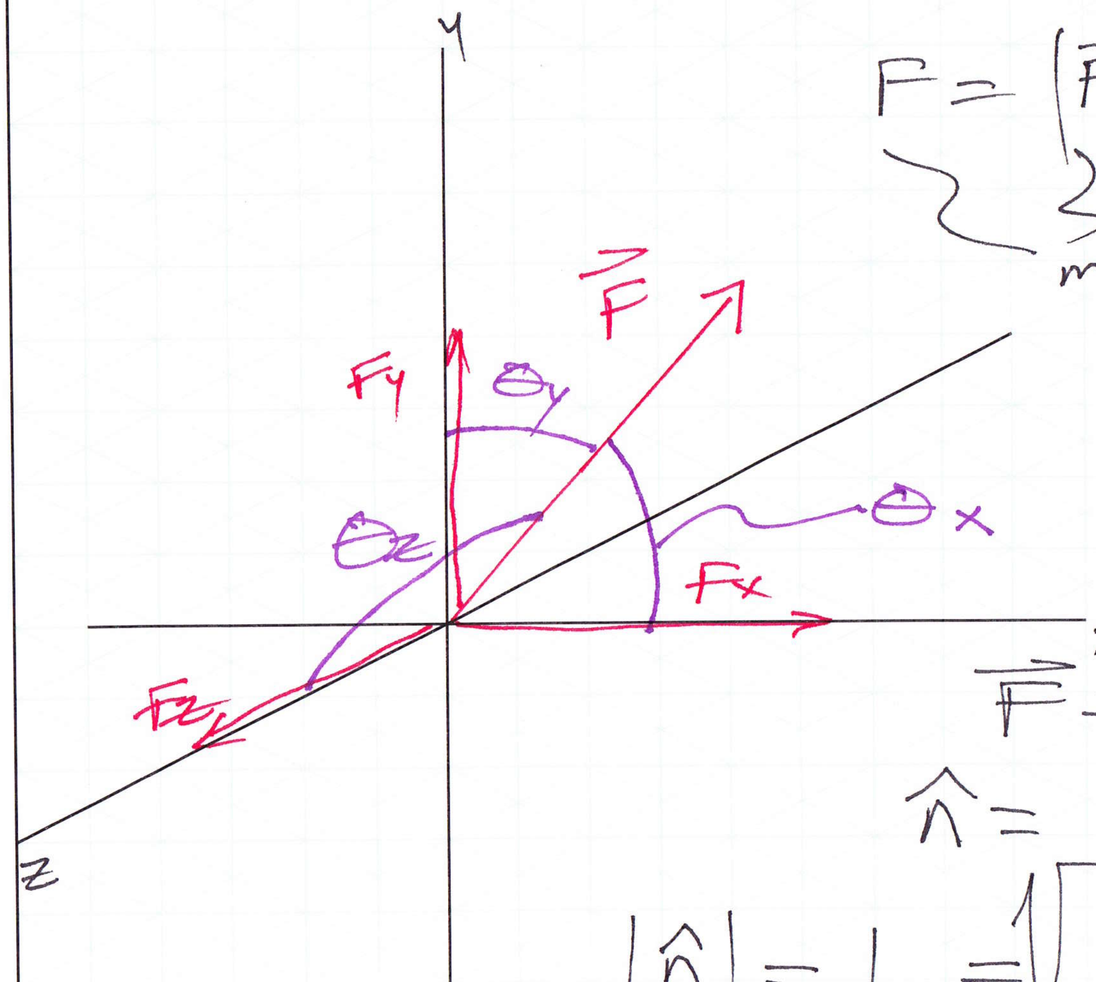
$$|\overrightarrow{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

Unit vector directed along $\overline{AB}$

$$\hat{n}_{AB} = \frac{(x_B - x_A)}{v}\,\hat{i} + \frac{(y_B - y_A)}{v}\,\hat{j} + \frac{(z_B - z_A)}{v}\,\hat{k}$$

$$\vec{F} = F\left[\frac{(x_B - x_A)}{v}\right]\hat{i} + F\left[\frac{(y_B - y_A)}{v}\right]\hat{j} + F\left\{\frac{(z_B - z_A)}{v}\right\}\hat{k}$$

Scalars



$F = |\vec{F}|$ } magnitudes

Direction Cosine Approach

$$F_x = \cos\theta_x |\vec{F}|$$

$$F_y = \cos\theta_y |\vec{F}|$$

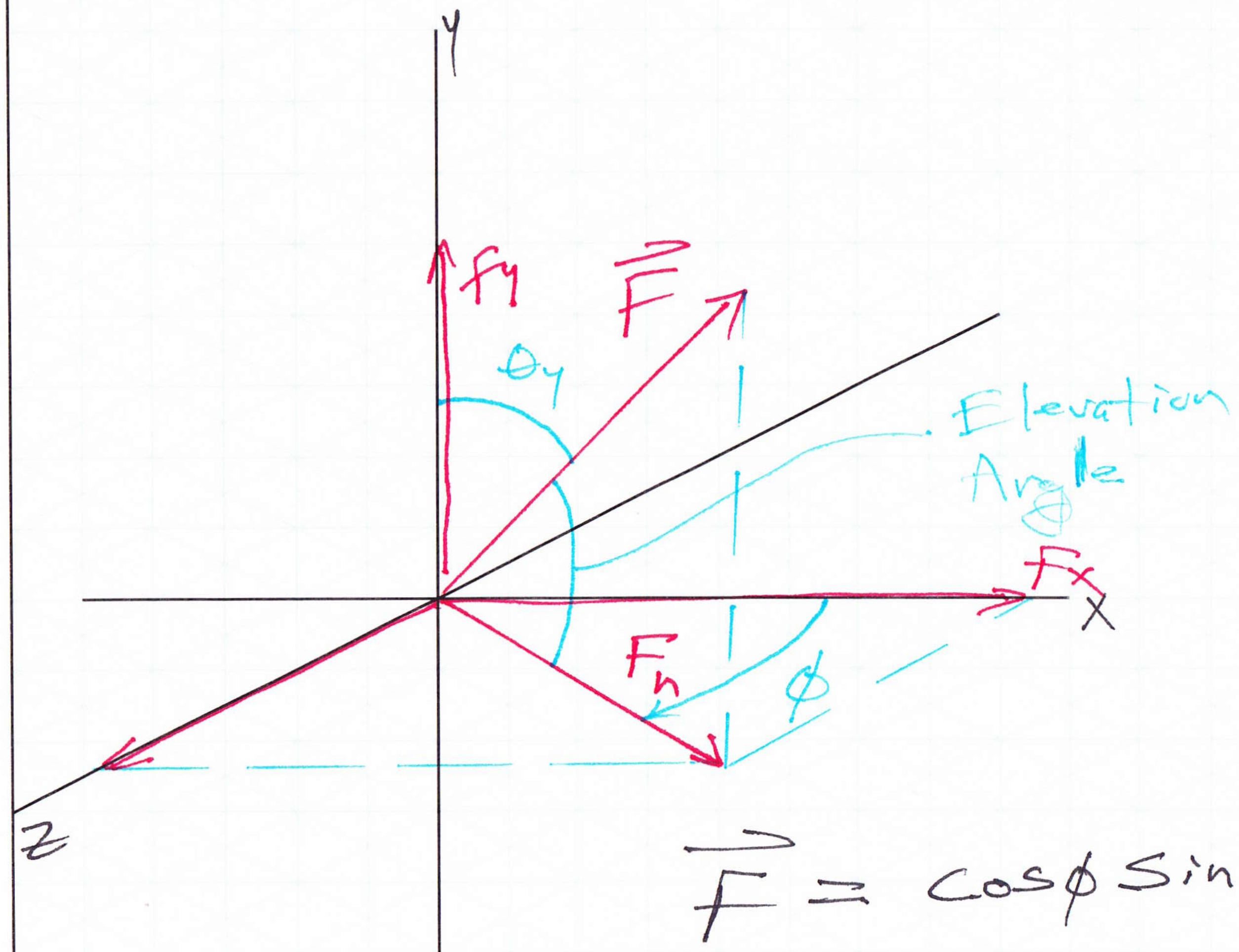
$$F_z = \cos\theta_z |\vec{F}|$$

$$\vec{F} = F\cos\theta_x \hat{i} + F\cos\theta_y \hat{j} + F\cos\theta_z \hat{k}$$

$$\hat{n} = \cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k}$$

$$|\hat{n}| = 1 = \sqrt{(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2}$$

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1$$



Azimuth Elevation

$$F_y = F\cos\theta_y$$

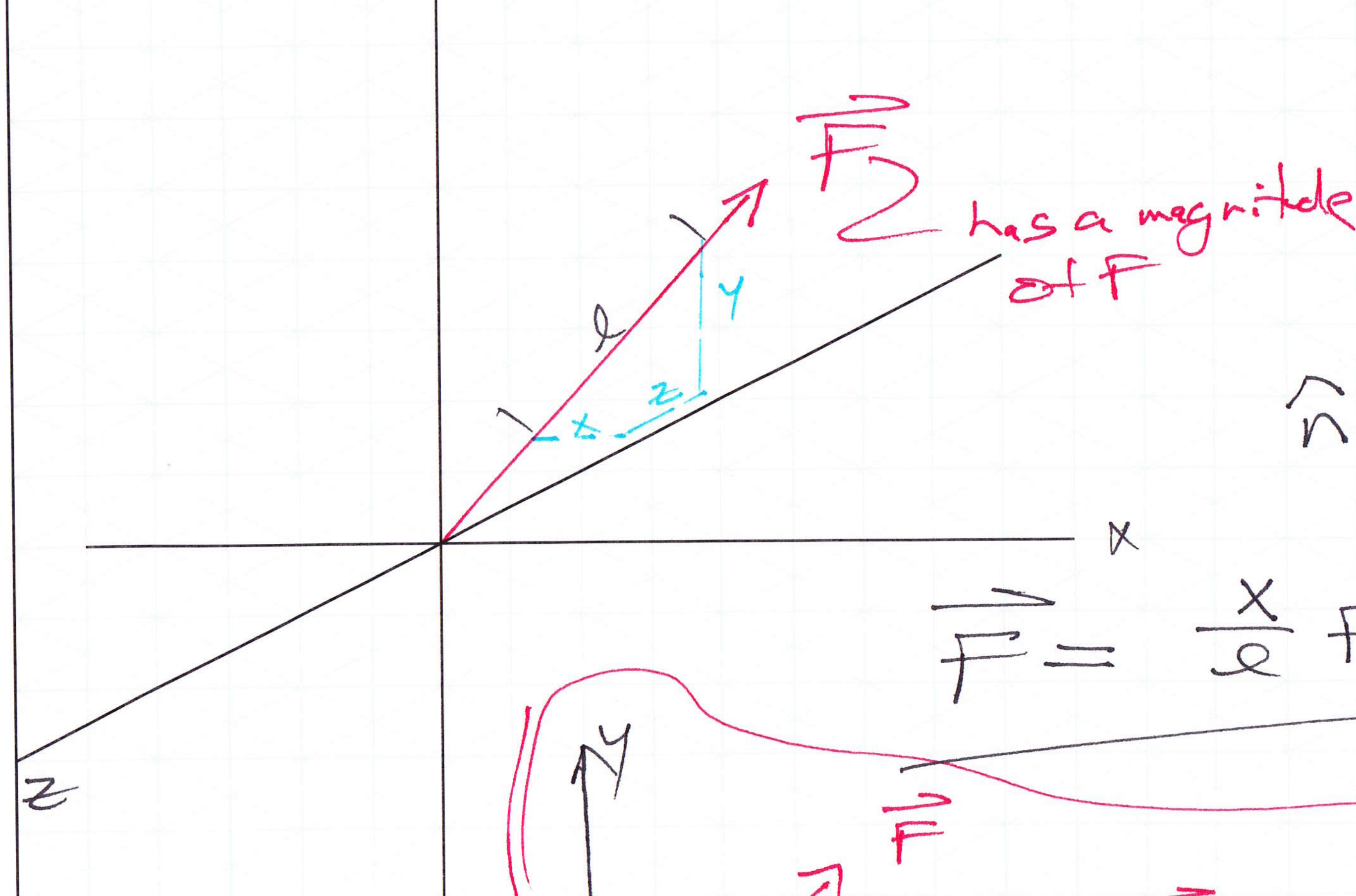
$$F_n = F\sin\theta_y$$

$$F_x = \cos\phi\, F\sin\theta_y$$

$$F_z = \sin\phi\, F\sin\theta_y$$

$$\vec{F} = \cos\phi\sin\theta_y F\,\hat{i} + \cos\theta_y F\,\hat{j} + \sin\phi\sin\theta_y F\,\hat{k}$$

x, y, z are projections



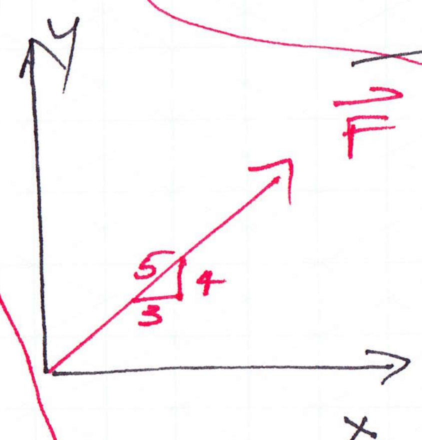
Slope Approach

$$l^2 = x^2 + y^2 + z^2$$

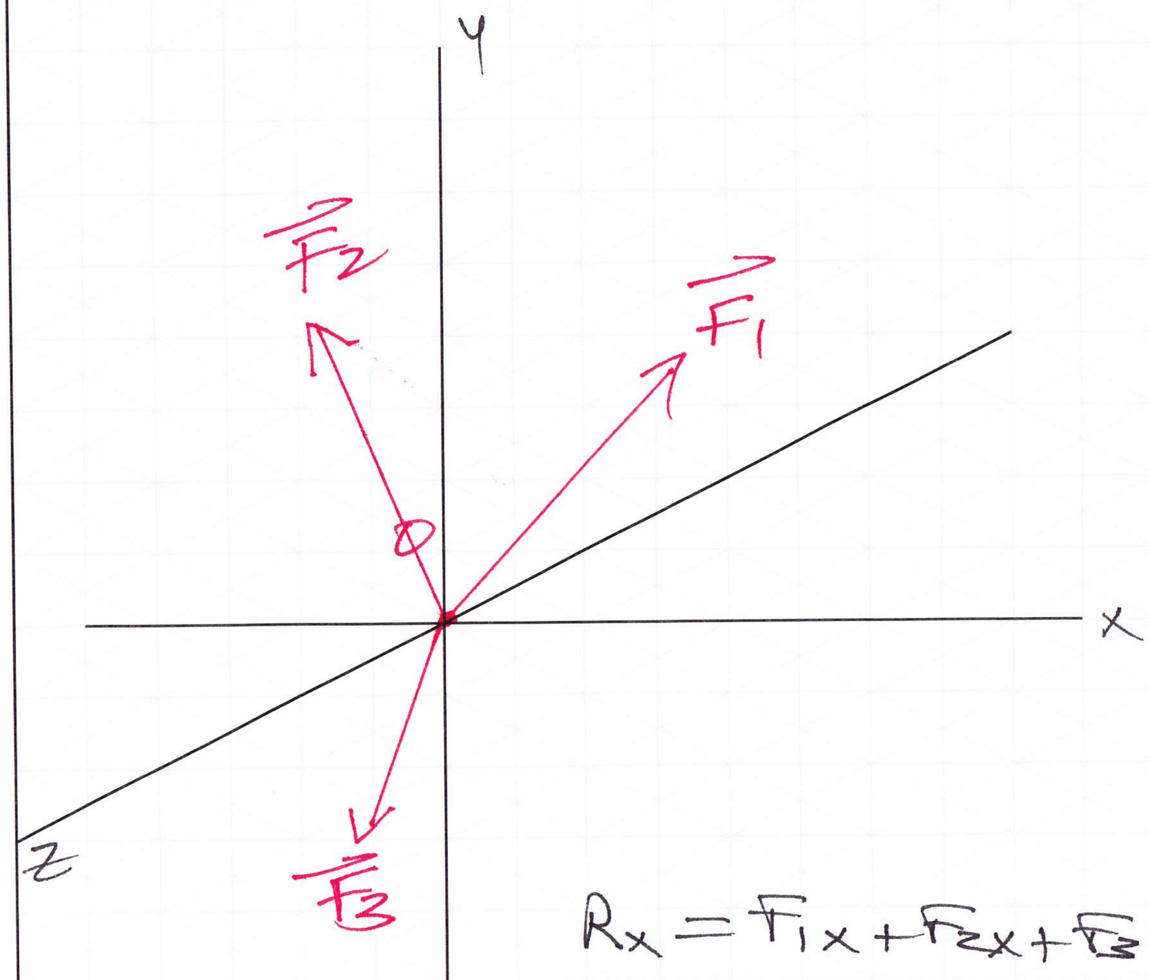
$$l = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{n} = \frac{x}{l}\hat{i} + \frac{y}{l}\hat{j} + \frac{z}{l}\hat{k}$$

$$\vec{F} = \frac{x}{l}F\hat{i} + \frac{y}{l}F\hat{j} + \frac{z}{l}F\hat{k}$$



$$\vec{F} = \frac{3}{5}F\hat{i} + \frac{4}{5}F\hat{j}$$



Addition of Concurrent 3-D Forces.

$$\vec{R} = \sum_{i=1}^{n} \vec{F}_i$$

$$\vec{F}_1 = F_{1x}\hat{i} + F_{1y}\hat{j} + F_{1z}\hat{k}$$

$$\vec{F}_2 = F_{2x}\hat{i} + F_{2y}\hat{j} + F_{2z}\hat{k}$$

$$\vec{F}_3 = F_{3x}\hat{i} + F_{3y}\hat{j} + F_{3z}\hat{k}$$

$R_x = F_{1x} + F_{2x} + F_{3x}$, $R_y = F_{1y} + F_{2y} + F_{3y}$, $R_z = F_{1z} + F_{2z} + F_{3z}$

$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$, $\cos\theta_x = \frac{R_x}{R}$, $\cos\theta_y = \frac{R_y}{R}$, $\cos\theta_z = \frac{R_z}{R}$

Equilibrium of a Particle

$$\vec{R} = 0$$

Scalar Equations of Equilibrium

$$\Sigma F_x = 0 \quad , \quad \Sigma F_y = 0 \quad , \quad \Sigma F_z = 0$$

3 equations which will allow us to solve for 3 unknowns.