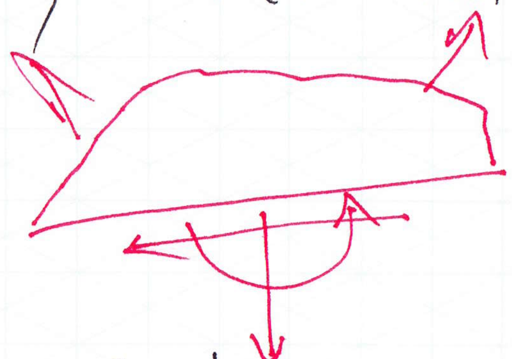
Rigid Bodies - Equivalent Systems of Forces

External Forces - action of other bodies on the rigid Body under consideration

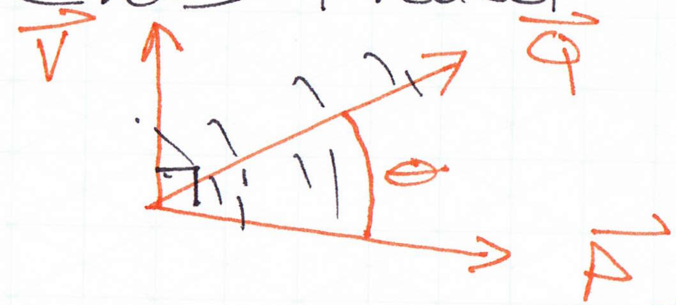
Internal Forces - the forces inside the rigid body that hold it together



Principle of transmissibility =

Vector Product of Two Vectors — Cross Product

$$\vec{V} = \vec{P} \times \vec{Q}$$



1. the line of action of $\vec{V}$ is $\perp$ to the plane defined by $\vec{P}$ and $\vec{Q}$

$\vec{P}$ and $\vec{Q}$ define a plane

2. the magnitude of $\vec{V}$ is given by

$$V = P\,Q\,\sin\theta$$

mag of V
mag of $\vec{P}$
angle between $\vec{P}$ and $\vec{Q}$
mag of $\vec{Q}$

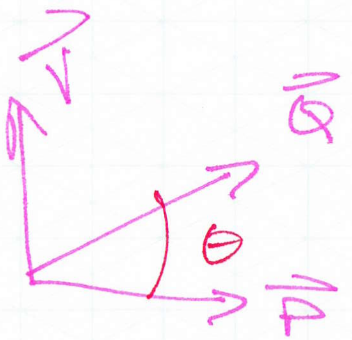
$\vec{V}$ is perpendicular to the plane defined by $\vec{P}$ and $\vec{Q}$

3. The direction of $\vec{V}$ is perpendicular to the plane containing $\vec{P}$ and $\vec{Q}$ and it orientation (sign) is obtained from the right hand rule.

$$\vec{P} \times \vec{Q} = -(\vec{Q} \times \vec{P})$$

Cross Product in Rectangular Components

$$\vec{V} = \vec{P} \times \vec{Q}$$



$$\vec{P} = P_x\,\hat{i} + P_y\,\hat{j} + P_z\,\hat{k}$$

$$\vec{Q} = Q_x\,\hat{i} + Q_y\,\hat{j} + Q_z\,\hat{k}$$

By definition

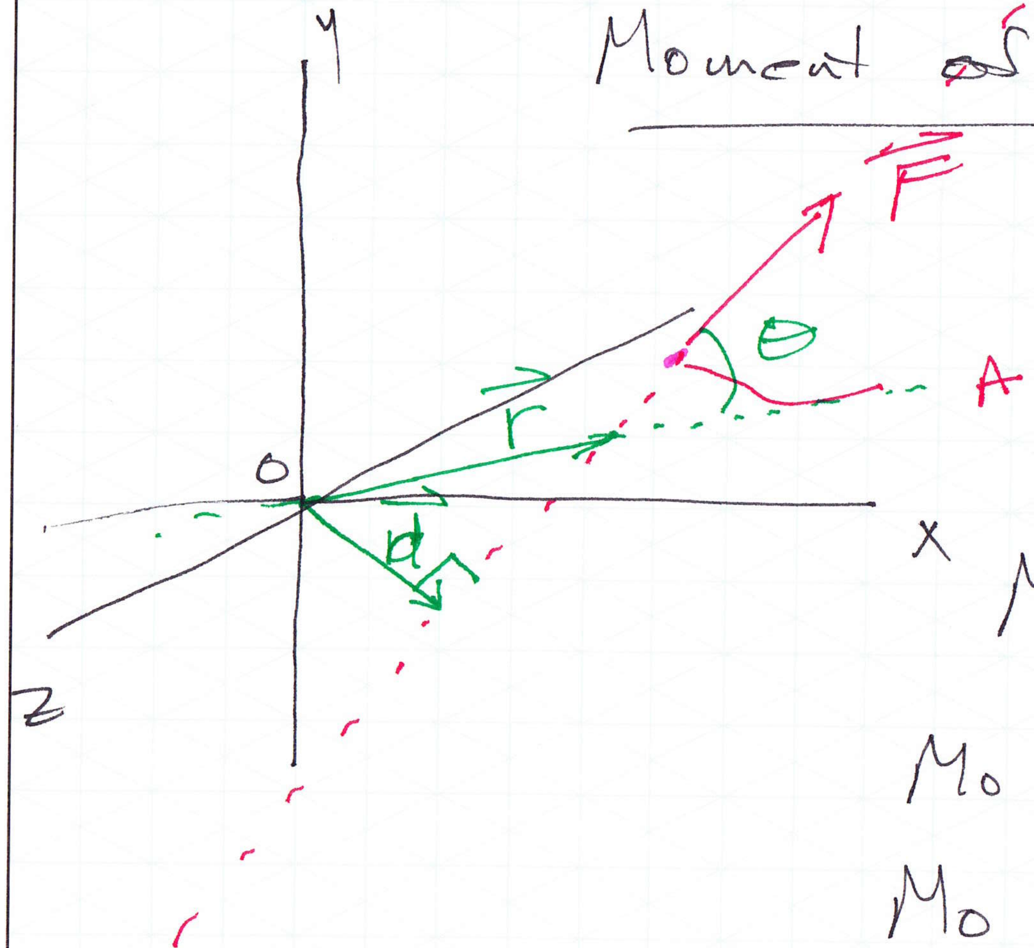
$$\vec{V} = \vec{P} \times \vec{Q} = \begin{array}{ccc} + & - & + \end{array} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

determinate

$$\vec{V} = +\underbrace{(P_y Q_z - Q_y P_z)}_{\text{scalar}}\,\hat{i} - \underbrace{(P_x Q_z - Q_x P_z)}_{\text{scalar}}\,\hat{j} + \underbrace{(P_x Q_y - Q_x P_y)}_{\text{scalar}}\,\hat{k}$$

Moment of a Force about a Point O

$$\vec{M}_O = \underbrace{\vec{r}}_{\text{position vector that extends from point O to any point along the line of action of } \vec{F}} \times \underbrace{\vec{F}}_{\text{Force vector}}$$

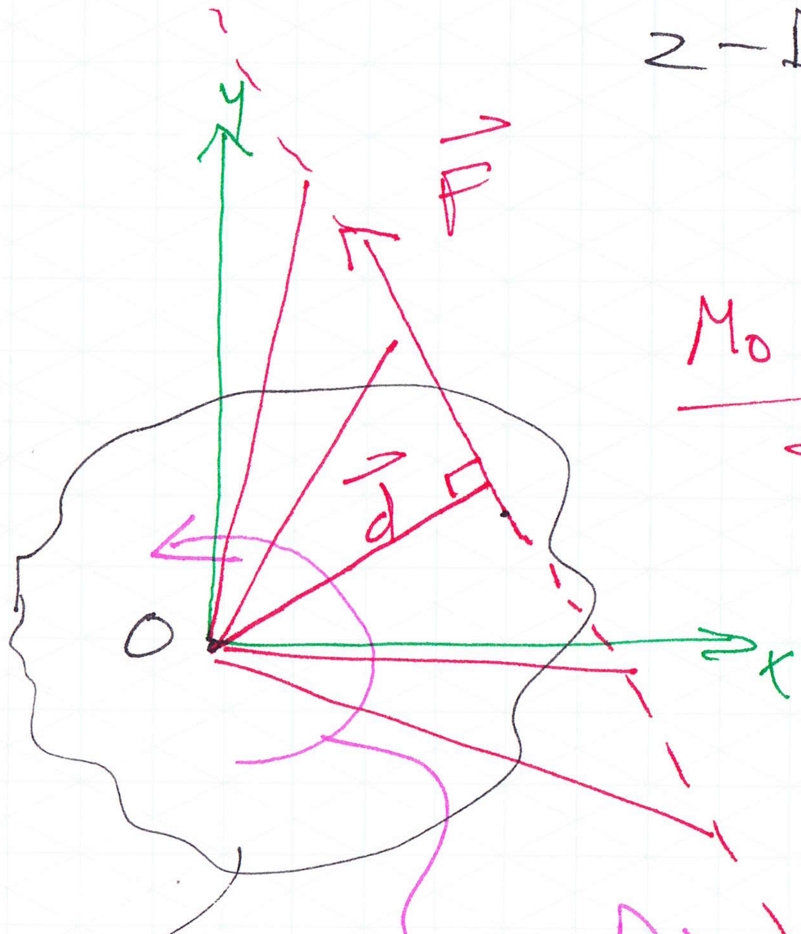


$$M_O = r F \sin\theta$$

$$M_O = d F \sin \cancel{\theta}^{\,90^\circ}$$

$$M_O = \underbrace{d}_{\text{scalar magnitude of the unique vector that extends from O to the line of action of } \vec{F}} F$$

2-D Moments

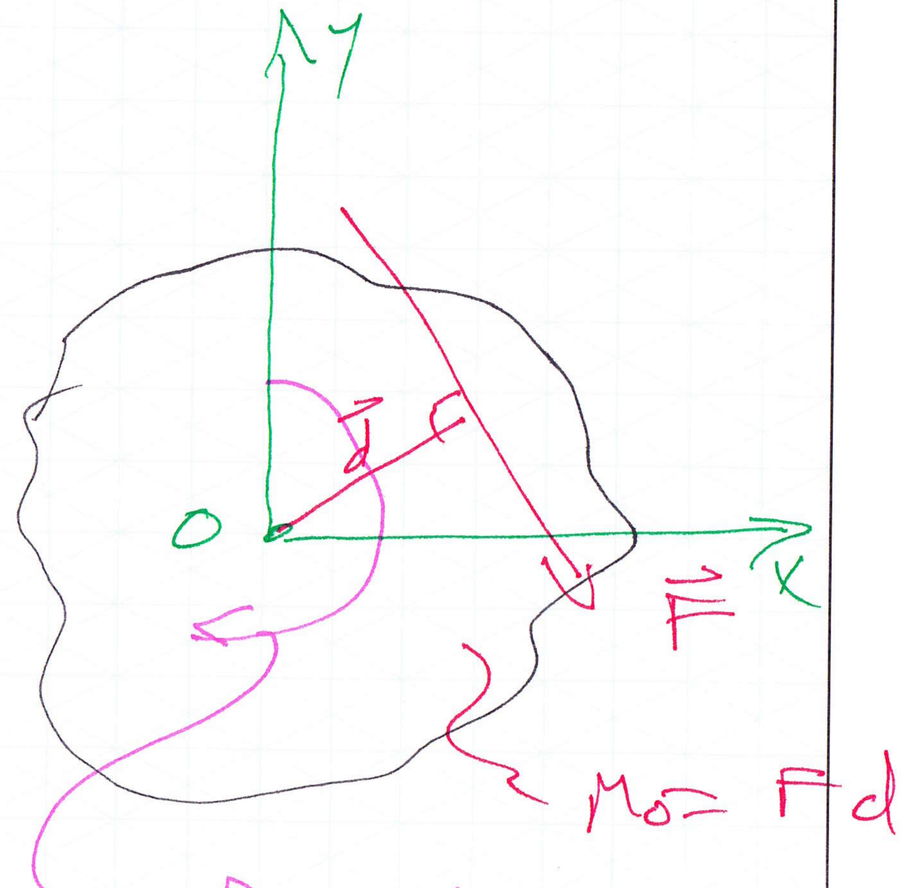


$M_O = Fd$

Direction is CCW

+

Planar 2-D rigid body

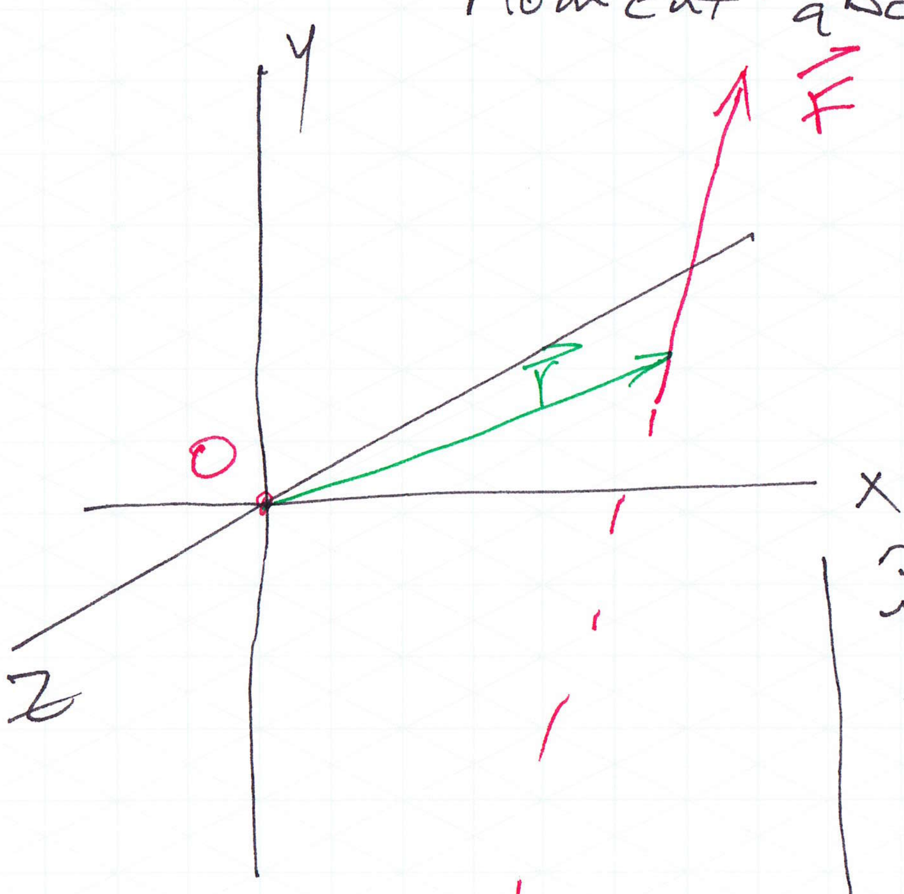


$M_O = Fd$

Direction is CW

−

Moment about a Point using rectangular Components



$$\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$$

$$\vec{F_x} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k}$$

$$\vec{M_O} = \vec{r} \times \vec{F}$$

$$\begin{vmatrix} \hat{i}^{+} & \hat{j}^{-} & \hat{k}^{+} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

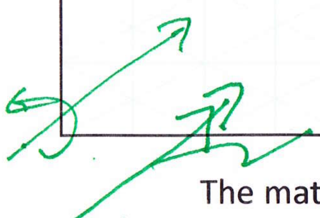
$\vec{M_O}$ – is a vector

You just use everthing you know about vectors

mag, direction cosines

etc.

$$\vec{M_O} = +(r_y F_z - F_y r_z)\,\hat{i} - (r_x F_z - F_x r_z)\,\hat{j} + (r_x F_y - F_x r_y)\,\hat{k}$$

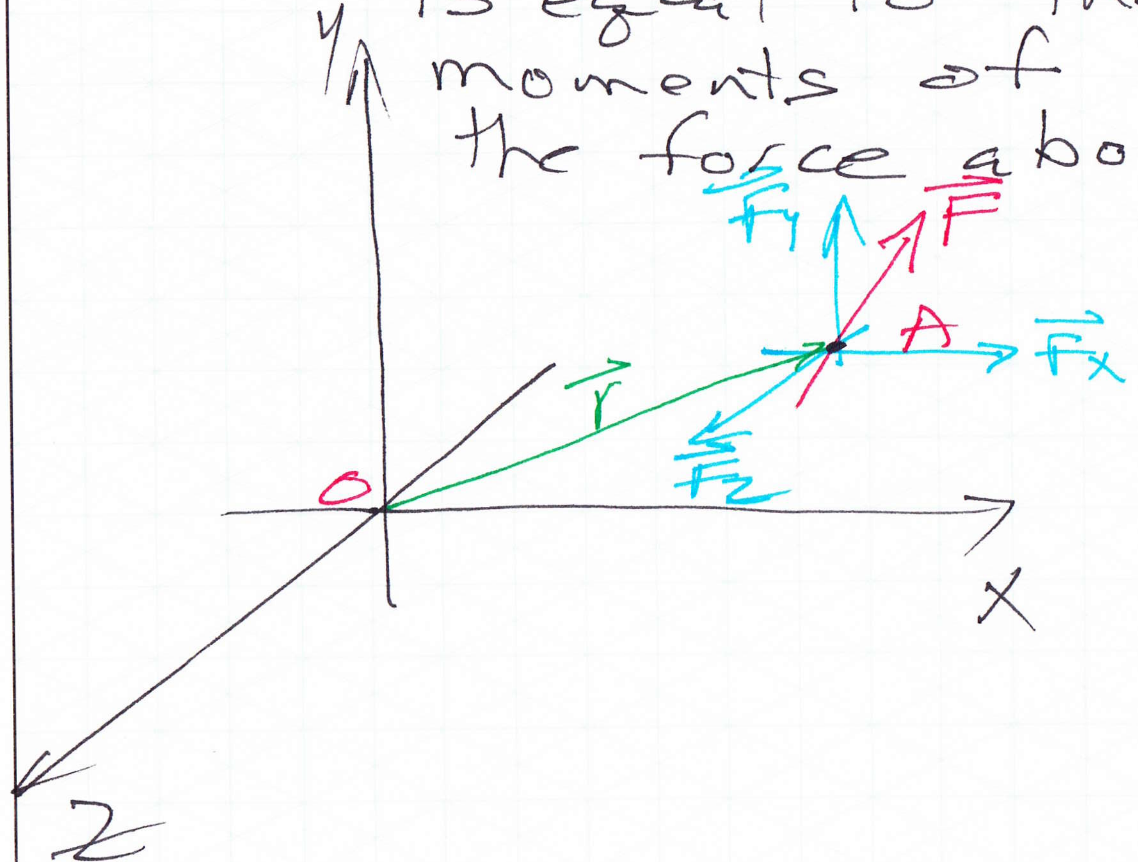


moments

Free Vector

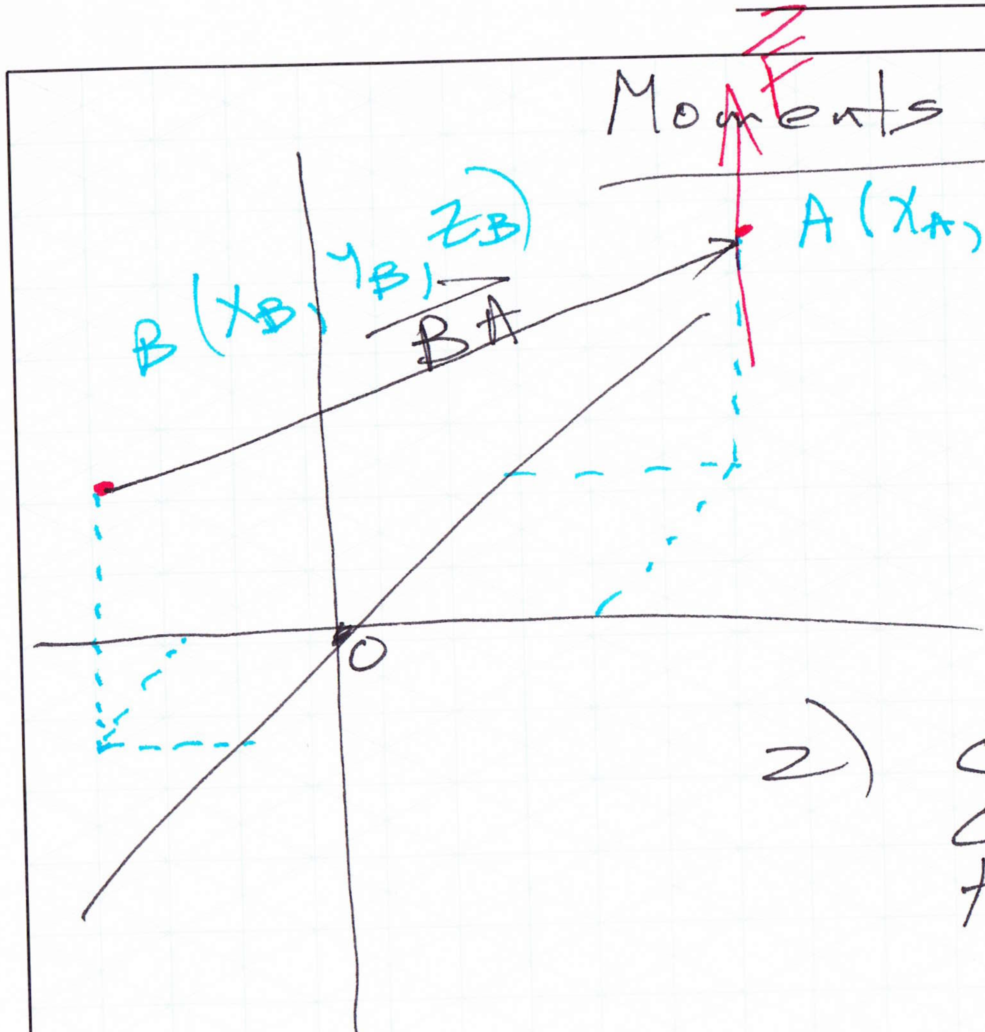
Varignon's Theorem

The moment of a force about a point is equal to the sum of the individual moments of all of the components of the force about the same point.



$$\vec{M}_O = \vec{r} \times \vec{F} = \vec{r} \times \vec{F}_x + \vec{r} \times \vec{F}_y + r \times \vec{F}_z$$

Moments of a Force about a general Point



Take the moment of the Force $\vec{F}$ applied at A about point B.

1) write $\vec{F}$ in rectangular components

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

2) choose a position vector that connects point B to any point along the line of action of $\vec{F}$

$$\overrightarrow{BA} = (x_A - x_B)\hat{i} + (y_A - y_B)\hat{j} + (z_a - z_B)\hat{k}$$

3) $\vec{M}_{F\,about\,B} = \overrightarrow{BA} \times \vec{F}$