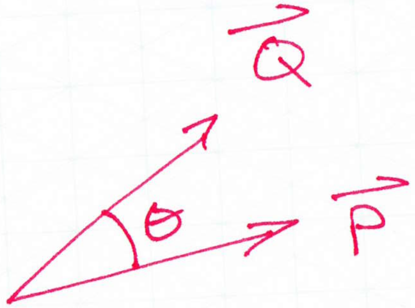


Scalar Product – Dot Product

$$\vec{P} \cdot \vec{Q} = |\vec{P}|\,|\vec{Q}| \cos\theta$$

this result is a scalar not a vector

Scalar Product is commutative

$$\vec{P} \cdot \vec{Q} = \vec{Q} \cdot \vec{P}$$

Scalar Product is distributive

$$\vec{P} \cdot (\vec{Q}_1 + \vec{Q}_2) = \vec{P} \cdot \vec{Q}_1 + \vec{P} \cdot \vec{Q}_2$$

Scalar Product for Rectangular Components

$$\vec{P} \cdot \vec{Q}$$

Assume that we have $\vec{P}$ and $\vec{Q}$ written in rectangular components

$$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

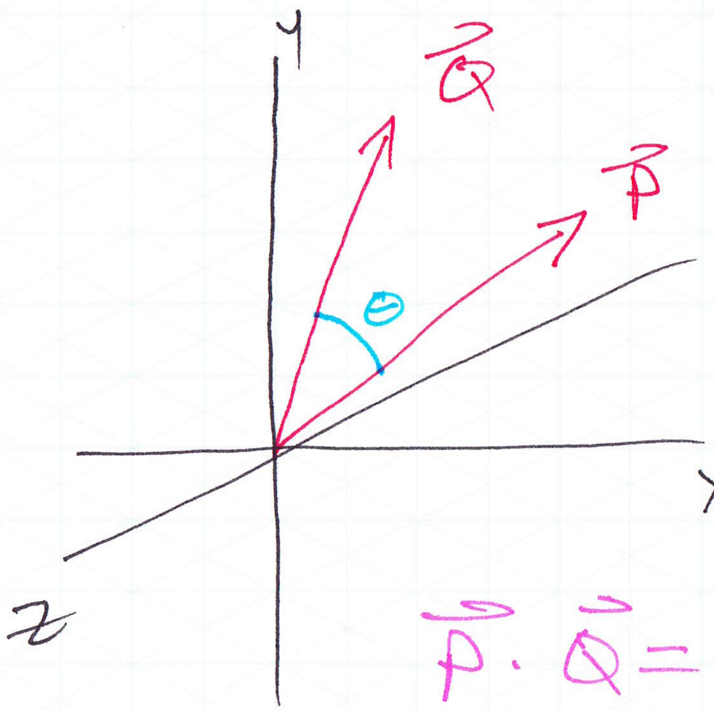
$$\vec{Q} = Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}$$

Then by definition —

$$\vec{P} \cdot \vec{Q} = P_x Q_x + P_y Q_y + P_z Q_z = |\vec{P}||\vec{Q}| \cos\theta$$

this sum is a single value

Two Primary Applications of the Scalar Product



Angle formed by two given vectors

$$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

$$\vec{Q} = Q_x \hat{i} + Q_y \hat{j} + Q_z \hat{k}$$

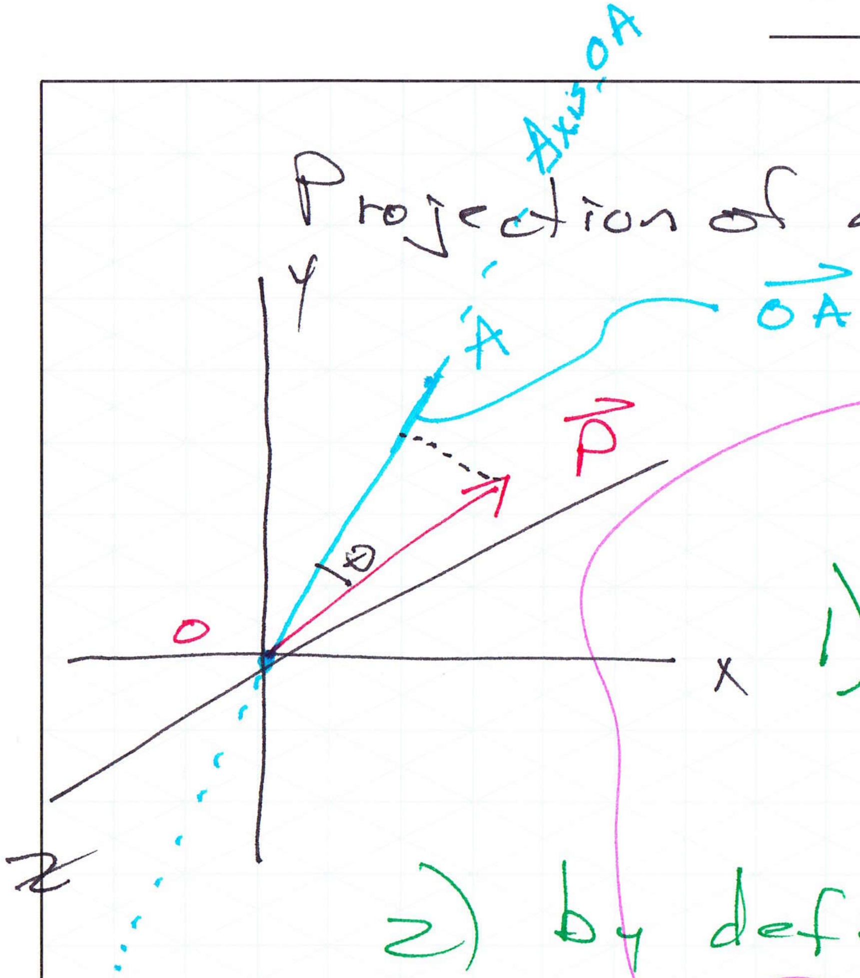
$$|\vec{P}| = \sqrt{P_x^2 + P_y^2 + P_z^2}, \quad |\vec{Q}| = \sqrt{Q_x^2 + Q_y^2 + Q_z^2}$$

$$\vec{P} \cdot \vec{Q} = |\vec{P}||\vec{Q}| \cos\theta = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\cos\theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{|\vec{P}|\,|\vec{Q}|}$$

inverse cos to find the angle.

Projection of a vector onto a given axis



The projection of $\vec{P}$ onto the OA axis

$$P_{OA} = |\vec{P}| \cos\theta$$

1) Establish a unit vector along the OA axis

$$\hat{n}_{OA} = n_{OAx}\hat{i} + n_{OAy}\hat{j} + n_{OAz}\hat{k}$$

2) by definition

$$P_{OA} = \vec{P} \cdot \hat{n}_{OA} = |\vec{P}|\cancel{|1|} \cos\theta$$

or

$$P_{OA} = P_x n_{OAx} + P_y n_{OAy} + P_z n_{OAz}$$

Mixed Triple Product

$$\vec{S} \cdot (\vec{P} \times \vec{Q})$$

Assume that we have everything expressed in rectangular components.

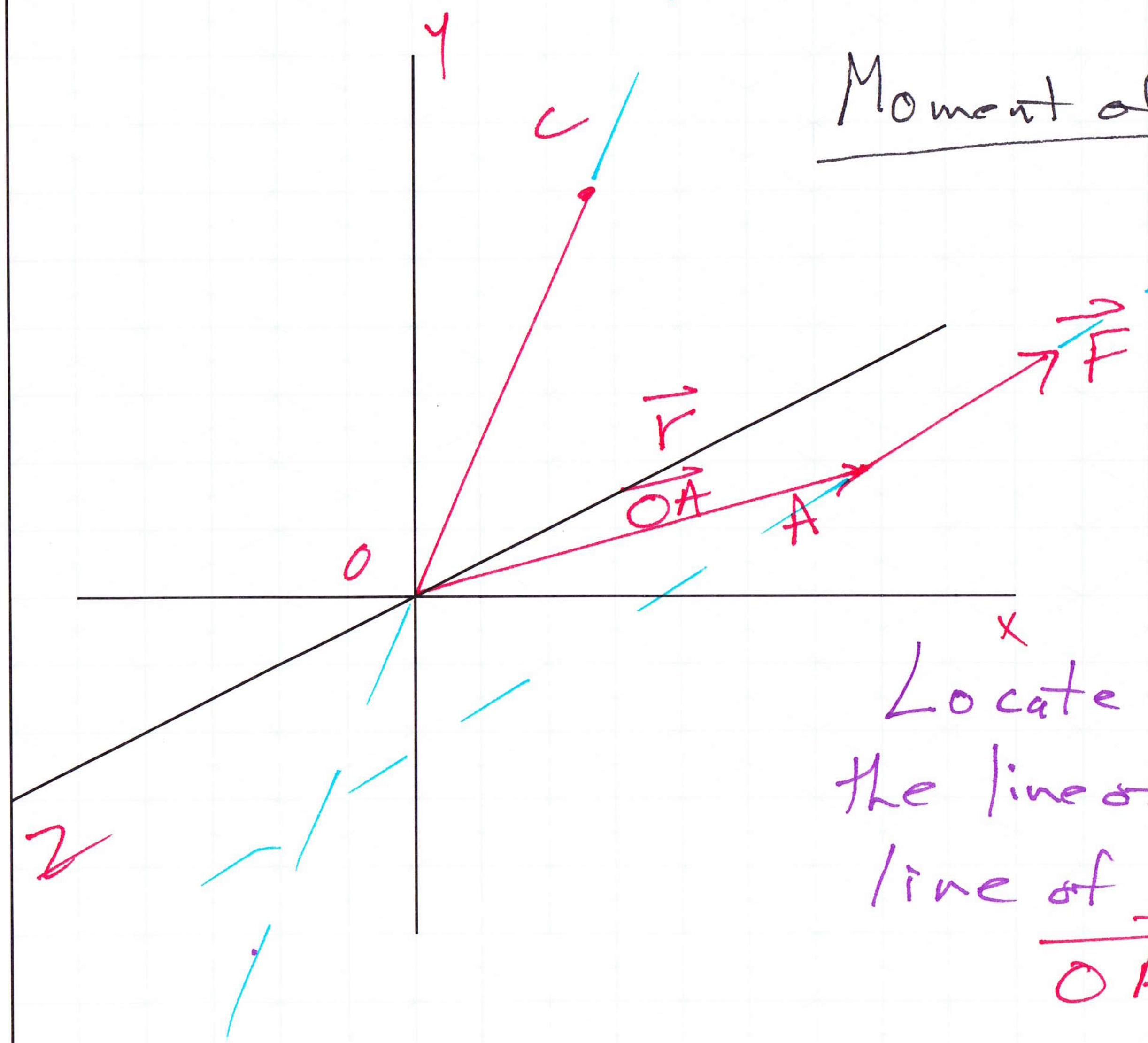
The result is a scalar

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = \begin{vmatrix} \overset{+}{S_x} & \overset{-}{S_y} & \overset{+}{S_z} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

$$\vec{S} \cdot (\vec{P} \times \vec{Q}) = +S_x (P_y Q_z - Q_y P_z) - S_y (P_x Q_z - Q_x P_z) + S_z (P_x Q_y - Q_x P_y)$$

$\vec{S} \cdot (\vec{P} \times \vec{Q}) =$ a single scalar term

magnitude only



Moment of a Force about a Given Axis

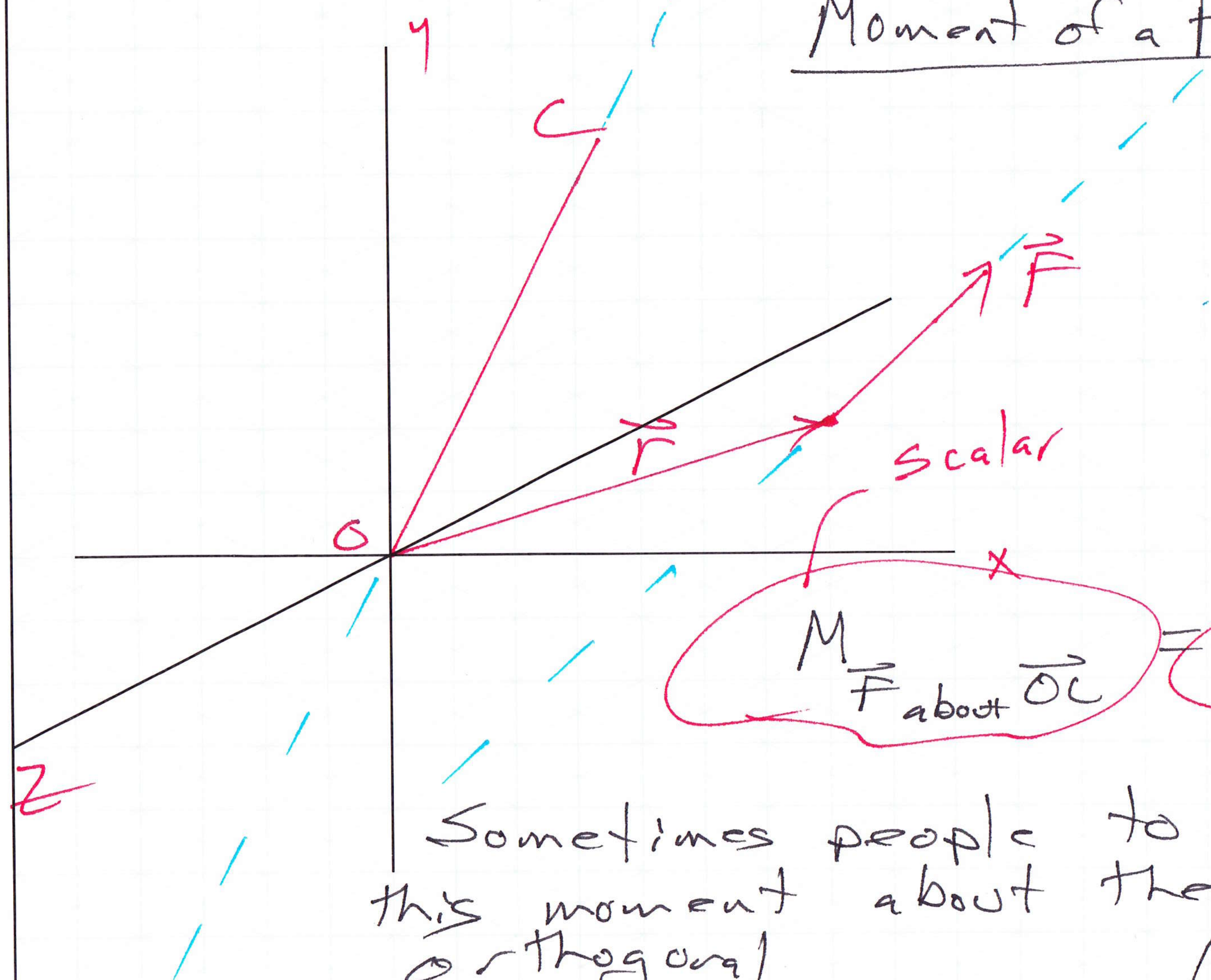
$\overrightarrow{OC}$ – defines the line of action of the given axis

$\vec{F}$ – force vector whose line of action passes through A

Locate a vector that connects the line of action of $\overrightarrow{OC}$ to the line of action of $\vec{F}$

$\overrightarrow{OA}$ $\vec{r}$

Moment of a Force about a Given Axis



$\vec{OC}$ – axis

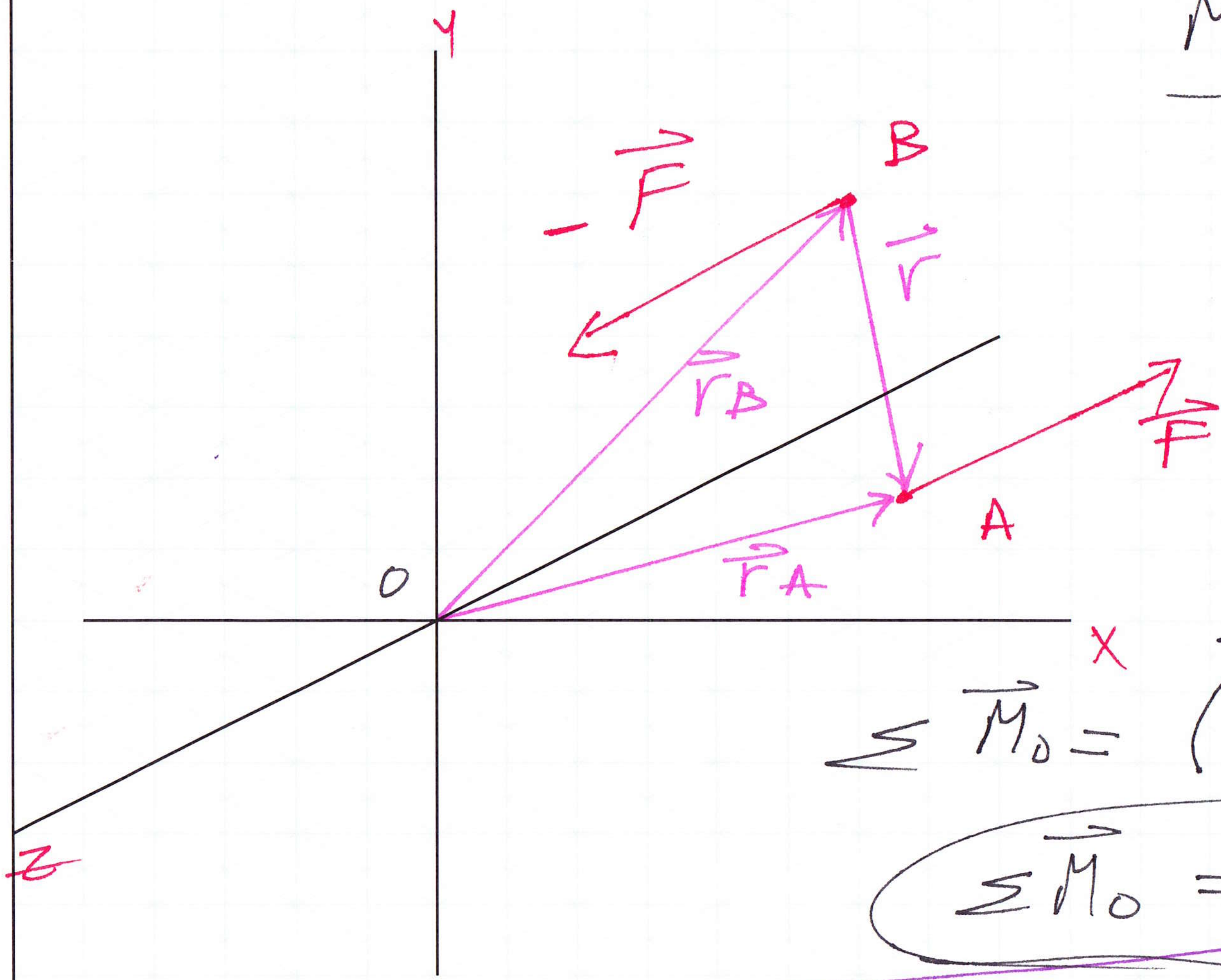
$\vec{F}$ – Force vector

$\vec{r}$ – connects the line of action of the axis and the line of action of the force

$$\underbrace{M_{\vec{F}\,about\,\vec{OC}}}_{\text{scalar}} = \underbrace{\hat{n}_{oc} \cdot (\vec{r} \times \vec{F})}_{\text{single value magnitude scalar}}$$

Sometimes people to express this moment about the axis in orthogonal

$$\vec{M}_{\vec{F}\,about\,\vec{OC}} = M_{\vec{F}\,about\,\vec{OC}} \left(n_{oc_x}\hat{i} + n_{oc_y}\hat{j} + n_{oc_z}\hat{k} \right)$$



Moment of a Couple

Two forces $\vec{F}$ and $-\vec{F}$ have the same magnitude, parallel lines of action, and opposite sense – Couple

$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

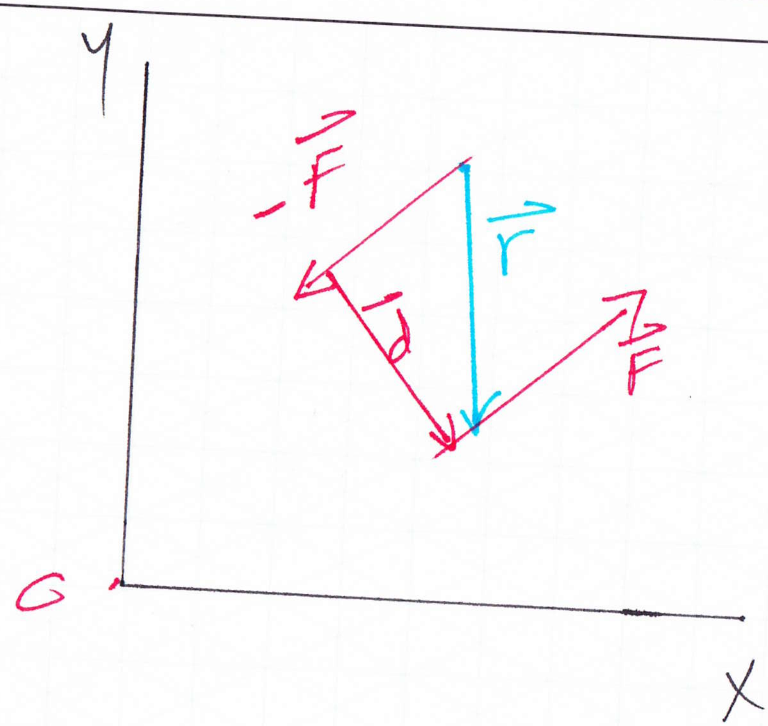
$$\Sigma \vec{M}_O = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F})$$

$$\Sigma \vec{M}_O = (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

$$\Sigma \vec{M}_O = \vec{r} \times \vec{F}$$

This is the moment of the couple and it acts ⊥ to the plane that contains $\vec{F}$ and $-\vec{F}$

Any vector that connects any point on the line of action of either force to any point on the line of action of the other force



$$\vec{M} = \vec{r} \times \vec{F}$$

$$|\vec{M}| = |\vec{r}|\,|\vec{F}| \sin\theta$$

Clever

we select $\vec{r}$ to be $\perp$ to the lines of action of the two forces then we say

$$|\vec{M}| = |\vec{d}|\,|\vec{F}| \cancel{\sin 90^\circ}$$

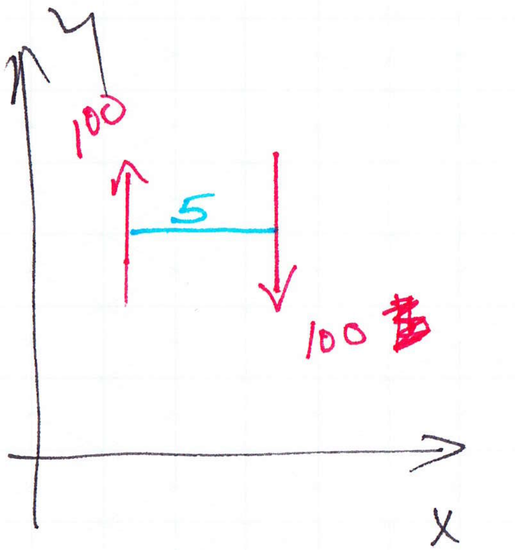
$$M = d\,F$$

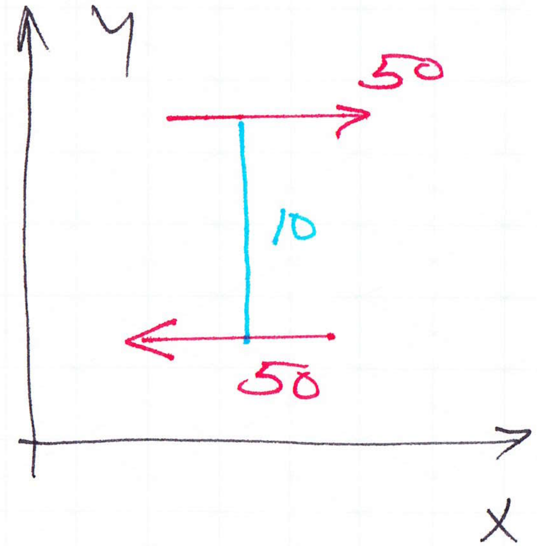
Magnitude of couple

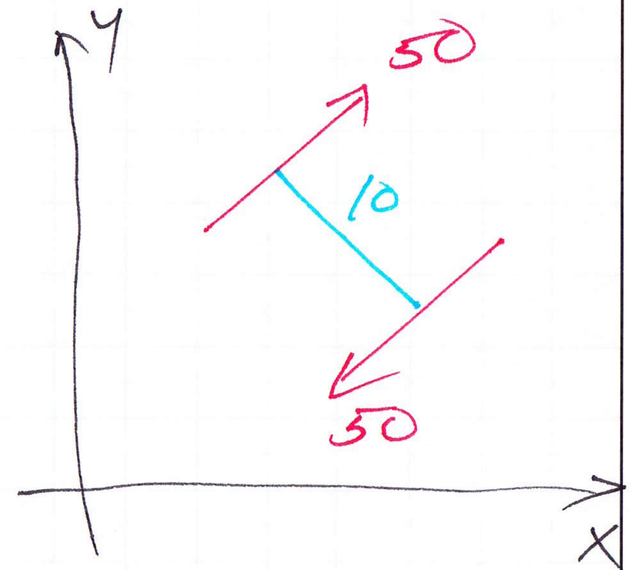
magnitude of the forces

magnitude $\perp$ distance between forces

Equivalent Couples







$$|\vec{M}| = (5)(100)$$
$$500$$

$$|\vec{M}| = (10)(50)$$
$$500$$

$$|\vec{M}| = (10)(50)$$
$$= 500$$

Couples are Moments — both are Vectors
cross product

Couples and Moments obey all of the rules of vectors that we have discussed.

All moment and couple vectors are Free Vectors