3-D Rigid Bodies

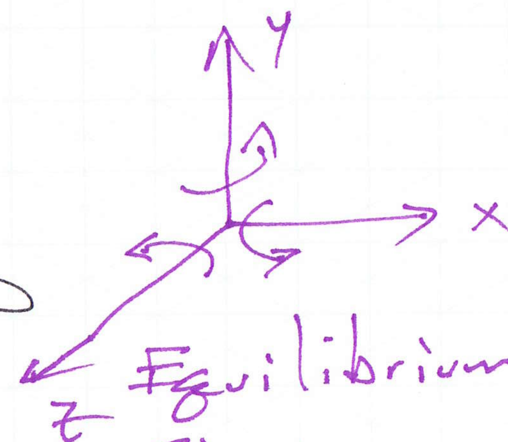
$$\sum \vec{F} = 0 \quad \text{and} \quad \sum \vec{M}_o = 0$$

Scalar Equations of Equilibrium

$\sum F_x = 0$ $\qquad$ $\sum M_x = 0$

$\sum F_y = 0$ $\qquad$ $\sum M_y = 0$

$\sum F_z = 0$ $\qquad$ $\sum M_z = 0$



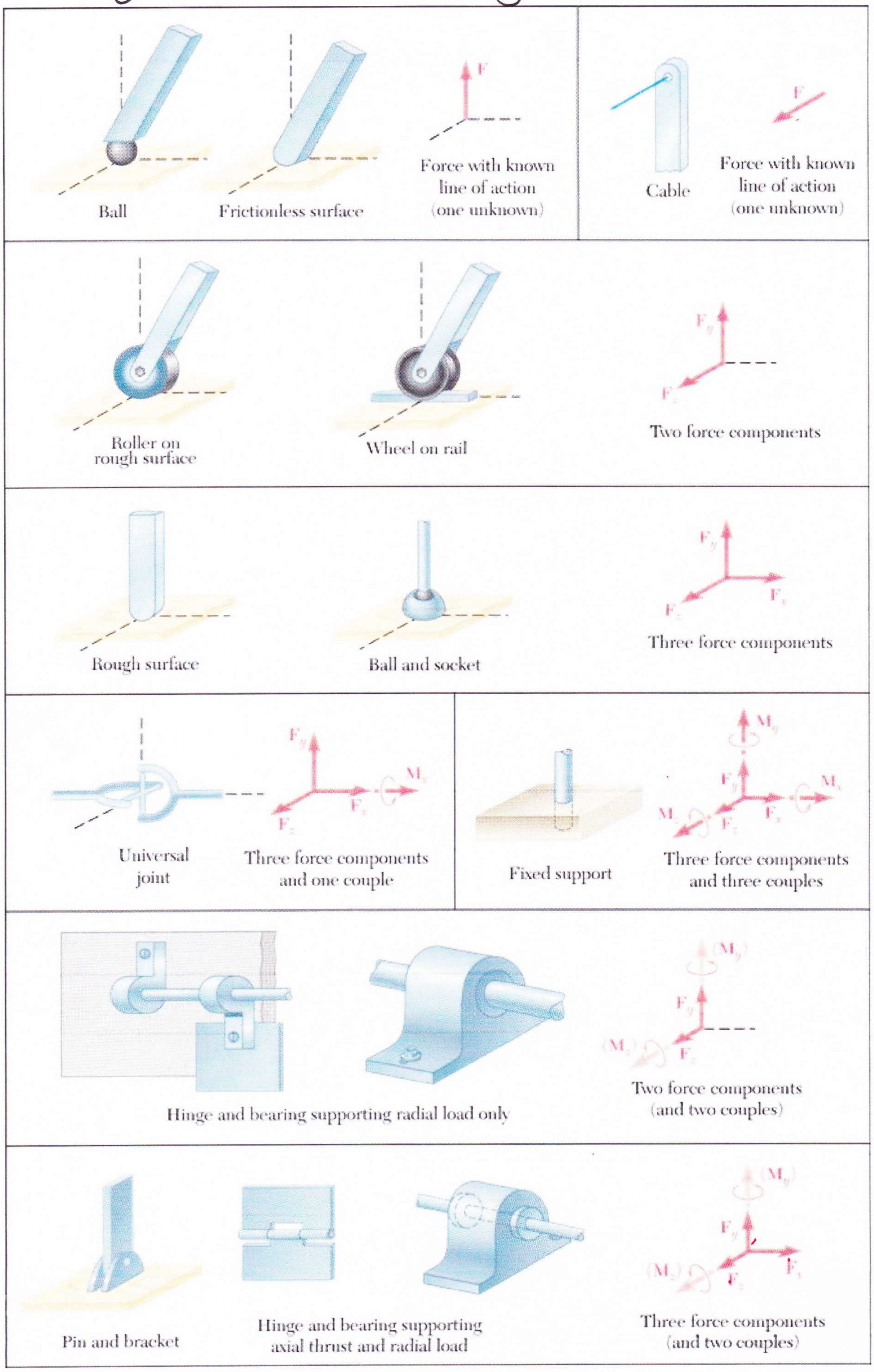
Equilibrium Sign Convention

$\vec{r} \times \vec{F}$

intuitively

Equilibrium of Rigid Bodies

2/14/14
2



3-D Rigid Bodies

F.B.D. – indicate forces (Known and unknown)

Apply scalar Equations of Equilibrium

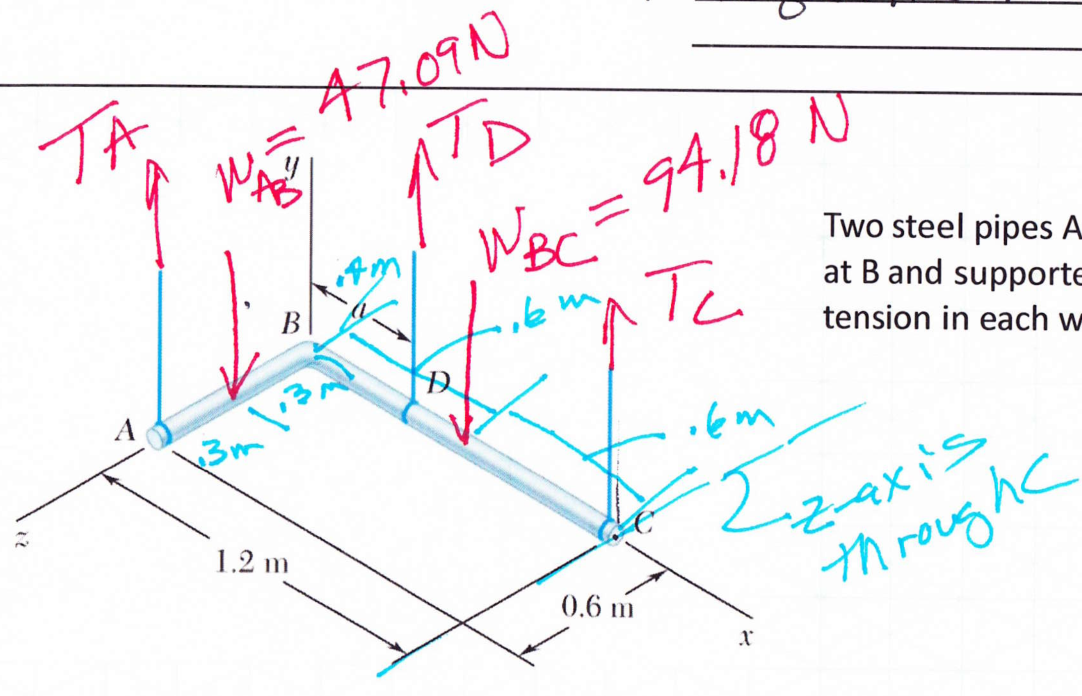
$\Sigma F_x = 0$ $\Sigma M_x = 0$

$\Sigma F_y = 0$ $\Sigma M_y = 0$

$\Sigma F_z = 0$ $\Sigma M_z = 0$

Sometimes we might Equations of conditions

Solve for the unknowns.



Two steel pipes AB and BC, each having a mass per unit length of 8 kg/m are welded together at B and supported by three vertical wires as shown. Knowing that a = 0.4 m, determine the tension in each wire.

$(8\ kg/m)(9.81\ m/s^2) = \underline{78.48\ N/m}$

$W_{BC} = 1.2(78.48) = \underline{94.18\ N}$

$W_{AB} = .6(78.48) = \underline{47.09\ N}$

F.B.D.

Equilibrium Sign Convention

1st observation

there are only 3 unknowns

3 < 6 - unstable

but because of the way that this problem is loaded - we can say that it is conditionally stable - we only need 3 equations

$\Sigma F_z = 0$ — useless

$\Sigma F_x = 0$ — useless

$\Sigma M_y = 0$ — useless

$\Sigma F_y = 0$, $\underbrace{M_z = 0}_{\text{any axis parallel to the z axis}}$, $\underbrace{\Sigma M_x = 0}_{\text{an axis parallel to the x axis}}$

Intuitive Approach

$$\Sigma M_{x\text{-axis}} = 0$$

$$-T_A(.6) + (47.09)(.3) = 0$$

$$T_A = 23.55\ N \quad \text{as shown}$$

$$\Sigma M_{z\text{-axis through } C} = 0$$

$$-\overset{23.55}{\cancel{T_A}}(1.2) + 47.09(1.2) - T_D(.8) + 94.18(.6) = 0$$

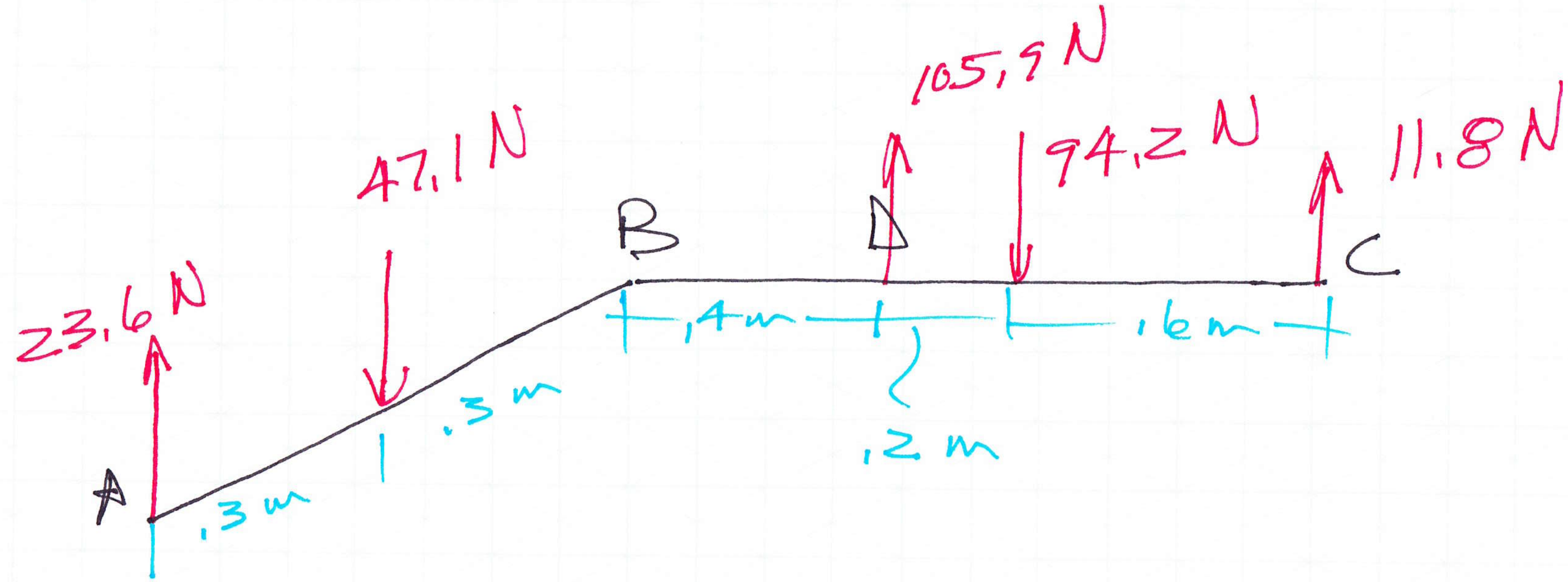
$$.8\,T_D = 84.76$$

$$\underline{T_D = 105.9\text{ N} \quad \text{as shown}}$$

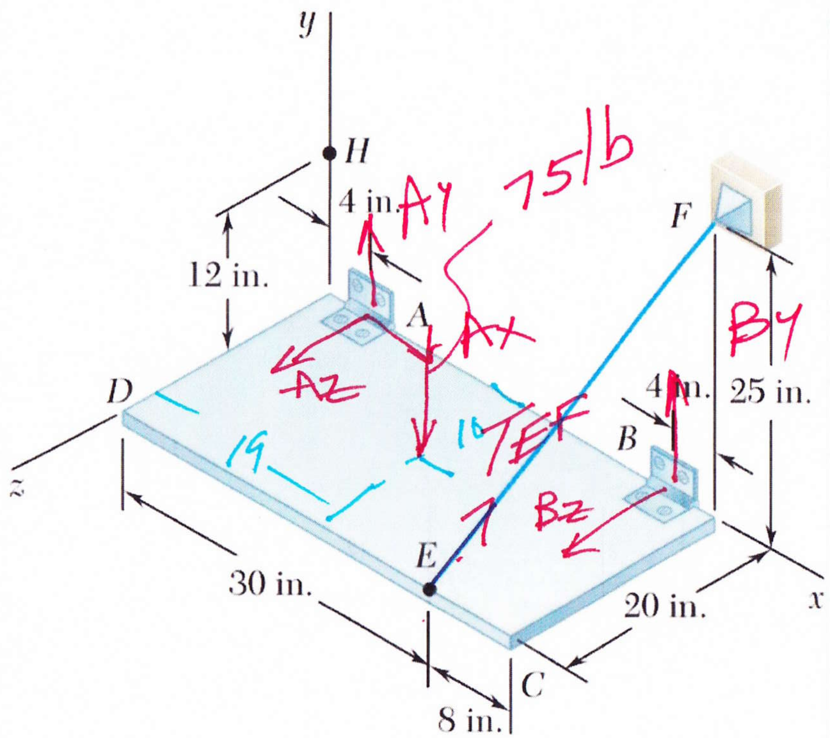
$$\uparrow \sum F_y = 0$$

$$\cancelto{23.55}{T_A} - 47.09 + \cancelto{105.9}{T_D} - 94.18 + T_C = 0$$

$$\underline{T_C = +11.82\ N} \quad \text{as shown}$$



Summary of Results



The uniform rectangular plate shown weighs 75 lb and is held in place as shown with hinges at A and B and by cable EF. Assume that the hinges do not induce moment reactions and that the hinge at B does not resist axial thrust. Determine the tension in cable EF.

$A_x, A_y, A_z, B_y, B_z, T_{EF}$

6 unknowns — 6 equations

stable — determinate

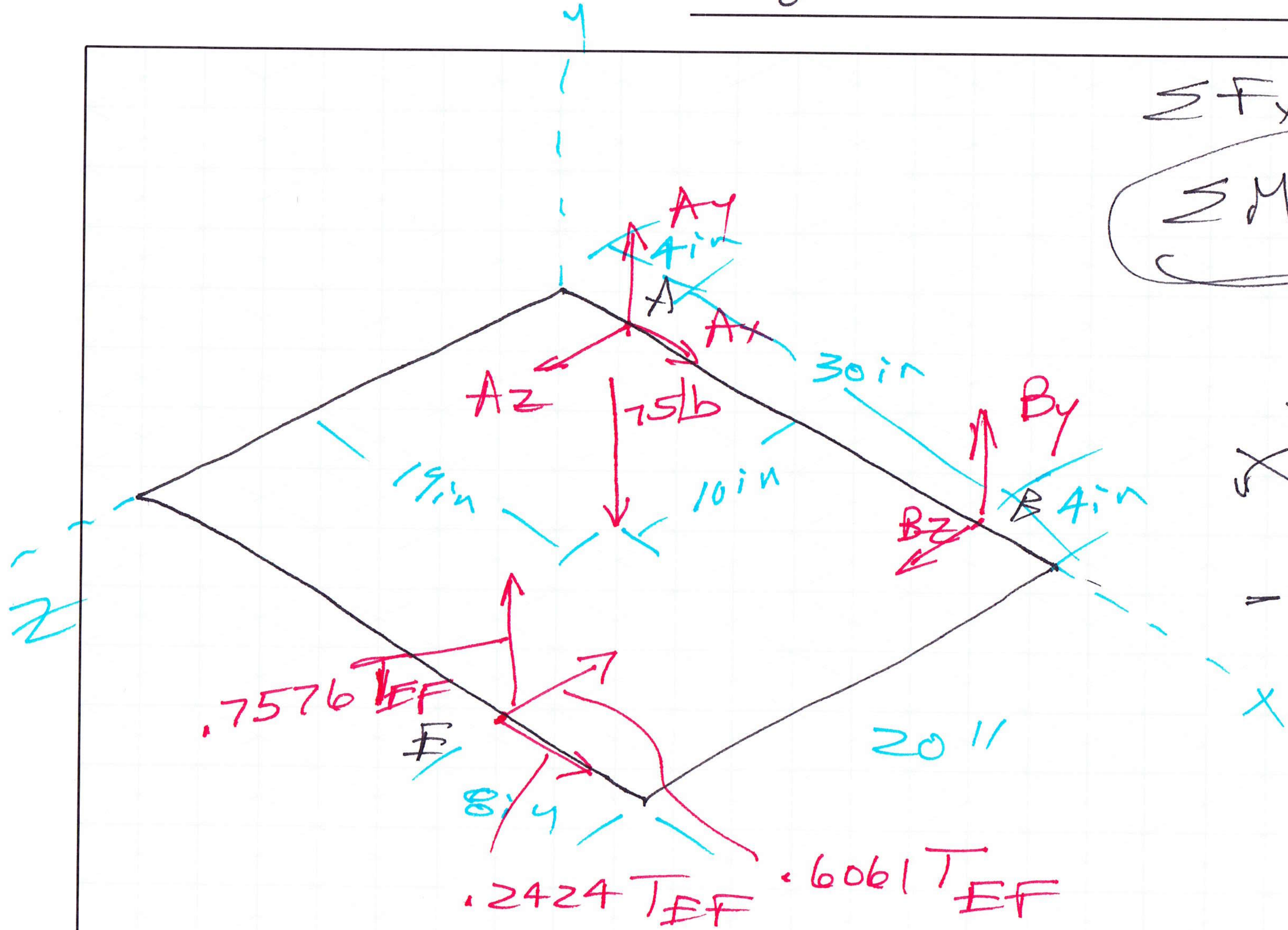
$$\overrightarrow{EF} = 8\hat{i} + 25\hat{j} - 20\hat{k}$$

$$|\overrightarrow{EF}| = \underline{33}$$

$$\hat{n}_{EF} = \frac{8}{33}\hat{i} + \frac{25}{33}\hat{j} - \frac{20}{33}\hat{k}$$

$$\hat{n}_{EF} = .2424\hat{i} + .7576\hat{j} - .6061\hat{k}$$

$$\overrightarrow{T}_{EF} = .2424T_{EF}\hat{i} + .7576T_{EF}\hat{j} - .6061T_{EF}\hat{k}$$



$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

$\Sigma M_x = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$

Intuitive Approach

$\Sigma M_x = 0$

$-(.7576\,T_{EF})(20) + 75(10) = 0$

$T_{EF} = 49.5\ lb$