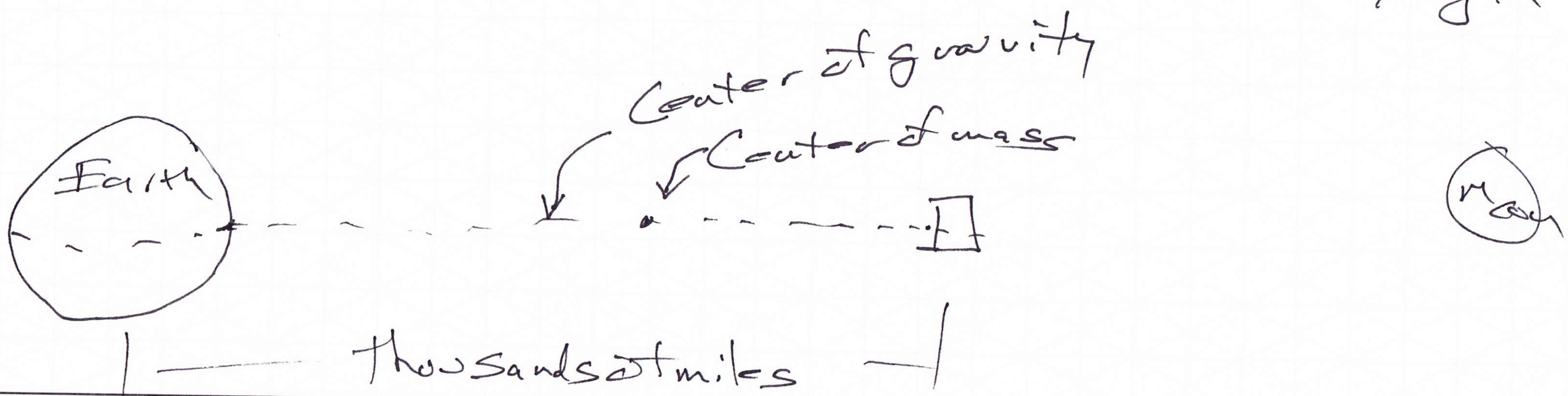
Topic: Centroids, Centers of Gravity, and Centers of Mass

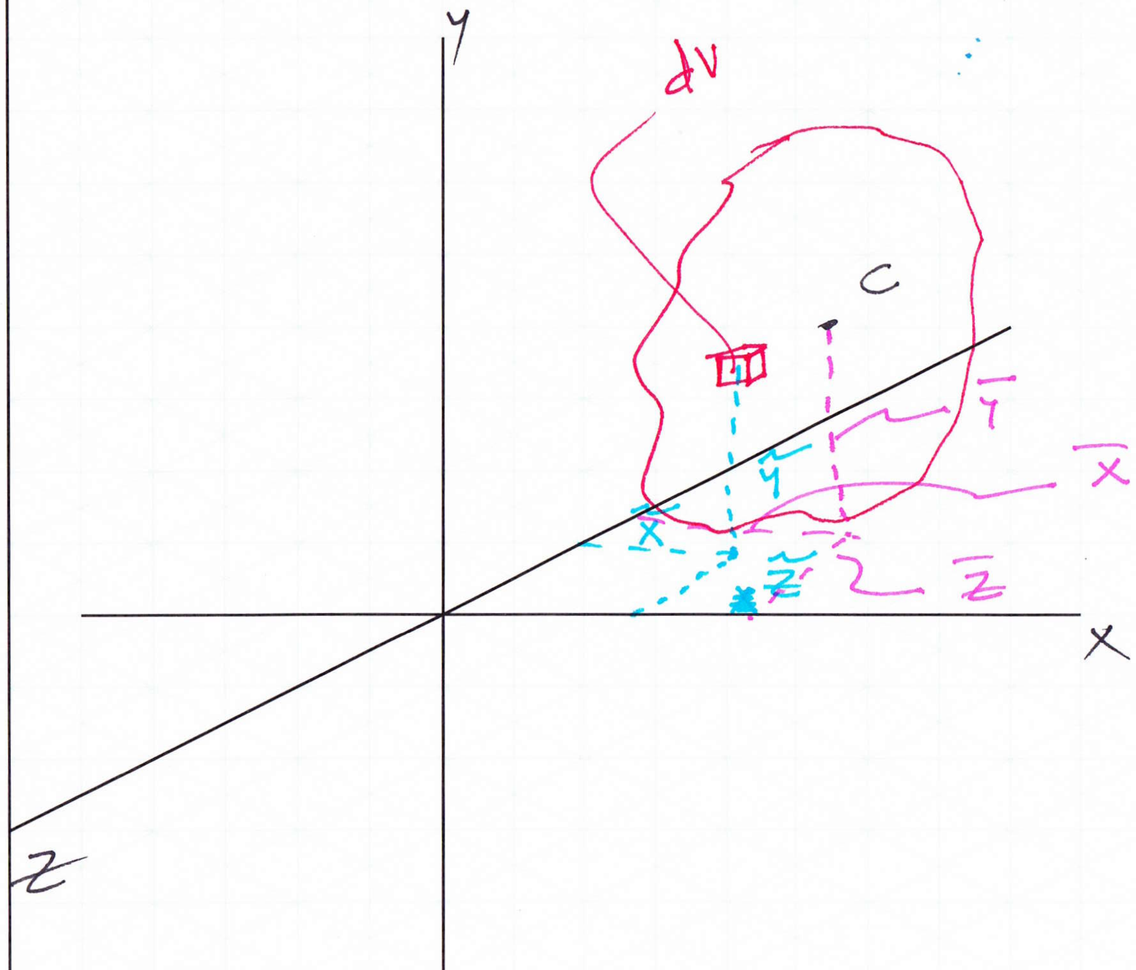
Date 2/16/14

— Center of gravity — weight is not independent of location — but as long as the object is small the effects of gravity will be uniform independent of location and size

— Center of Mass —

⇒ Centroids — geometric shape — volume, area, length





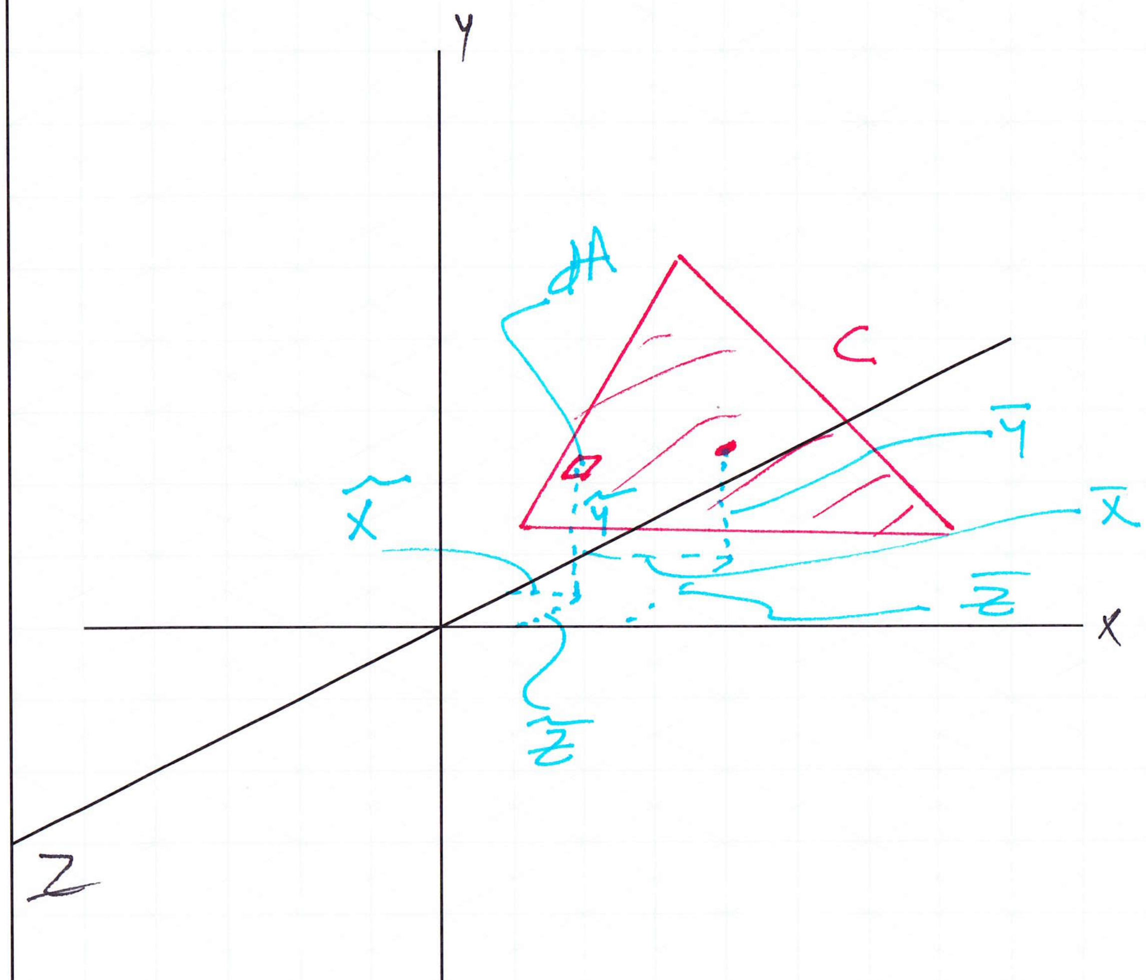
Centroid of a Volume

$$\bar{x} = \frac{\gamma \cancel{\rho} \int \tilde{x}\, dV}{\gamma \cancel{\rho} \int dV}$$

$$\bar{y} = \frac{\gamma \cancel{\rho} \int \tilde{y}\, dV}{\gamma \cancel{\rho} \int dV}$$

$$\bar{z} = \frac{\gamma \cancel{\rho} \int \tilde{z}\, dV}{\gamma \cancel{\rho} \int dV}$$

Center of Mass introduce desity term

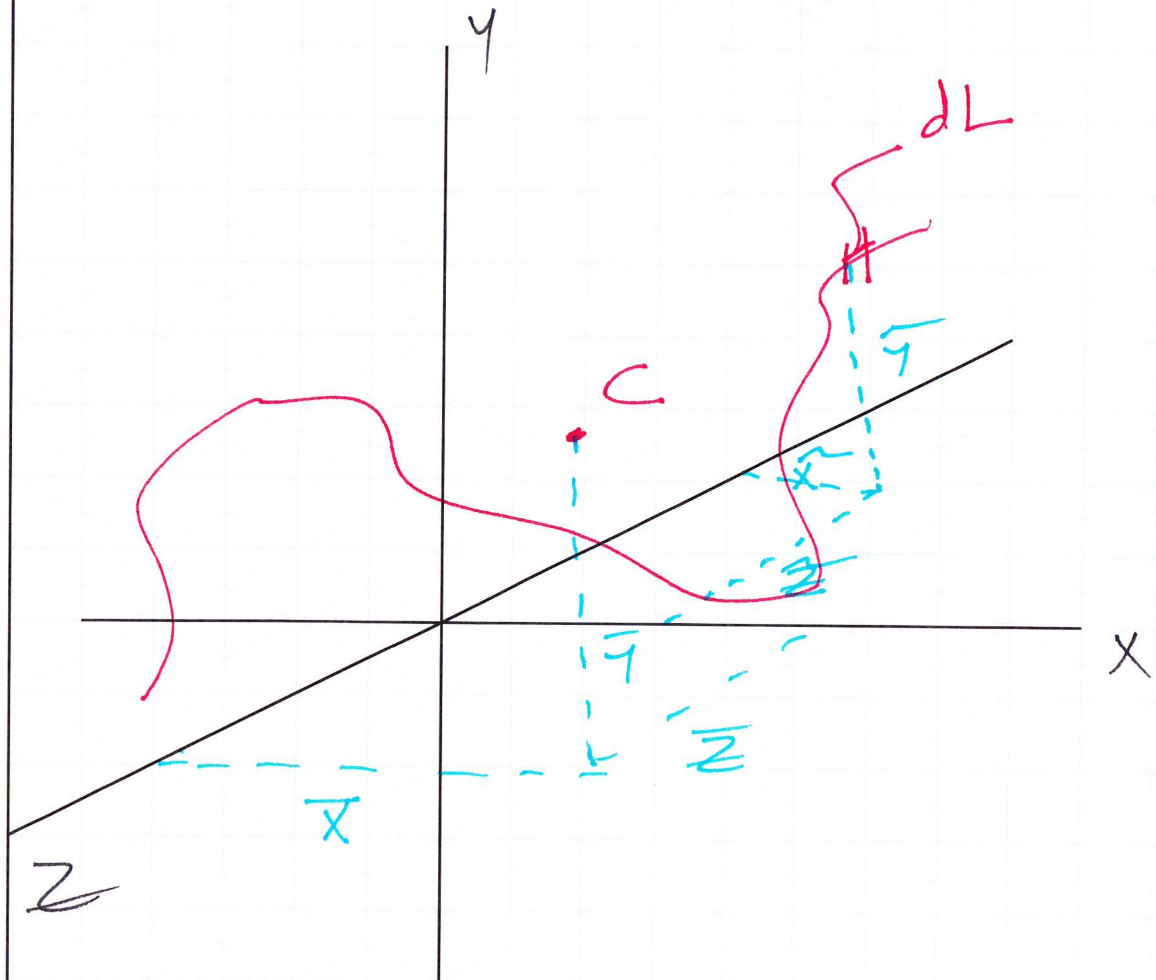


Centroid of an Area

$$\bar{x} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$$

$$\bar{z} = \frac{\int \tilde{z}\, dA}{\int dA}$$



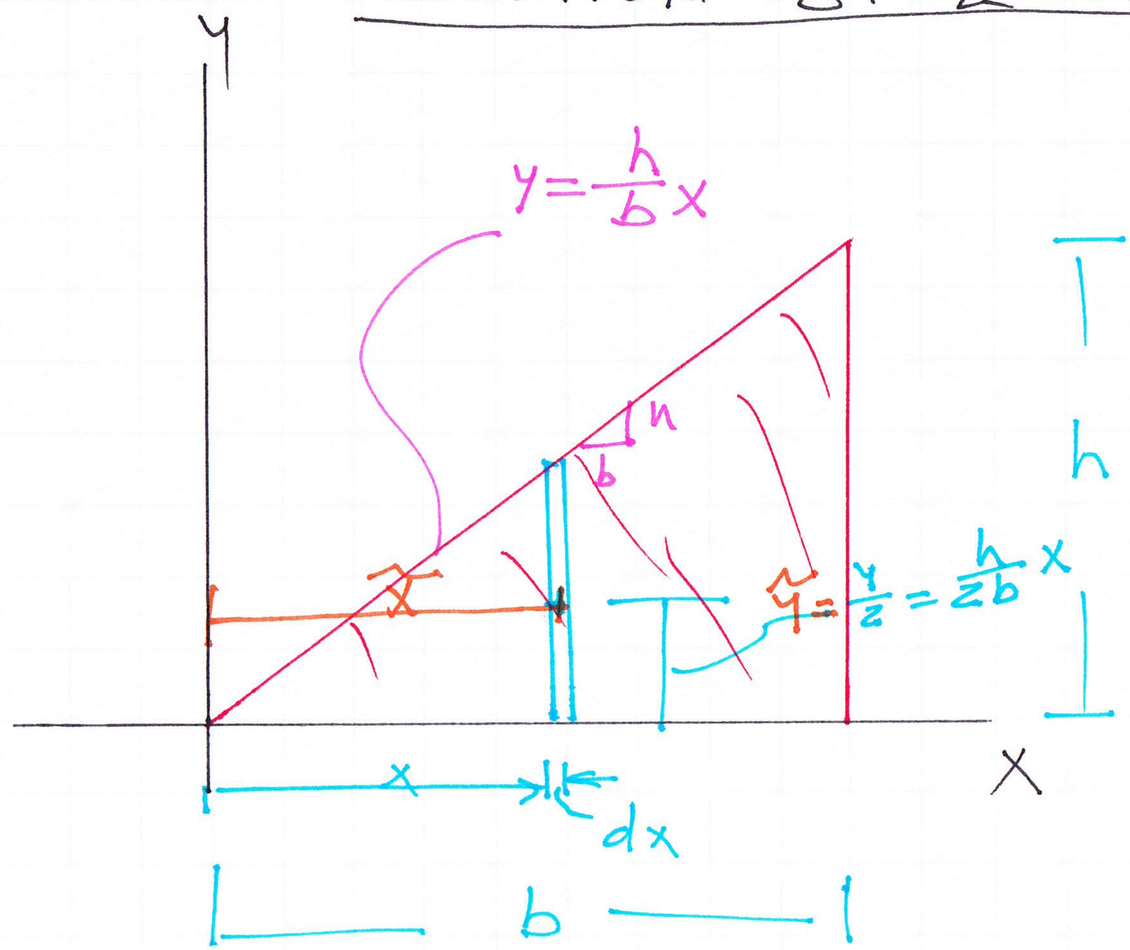
Centroid of a Line

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL}$$

$$\bar{y} = \frac{\int \tilde{y}\, dL}{\int dL}$$

$$\bar{z} = \frac{\int \tilde{z}\, dL}{\int dL}$$

Centroid of a Right Triangle



$$dA = \left(\frac{h}{b}x\right)dx$$

$$A = \int_0^b \frac{h}{b}x \, dx$$

$$A = \frac{h}{b}\int_0^b x \, dx$$

$$A = \frac{h}{b}\left[\frac{1}{2}x^2\right]\Bigg|_0^b$$

$$A = \frac{h\,b^{\cancel{2}}}{2\cancel{b}}$$

$$\boxed{A = \frac{bh}{2}}$$

$$z = 0, \quad \bar{x} = \frac{2}{3}b, \quad \bar{y} = \frac{1}{3}h$$

$\bar{X}$

$$\int \tilde{x}\, dA$$

$$\int_0^b x\left(\frac{h}{b}x\right)dx$$

$$\frac{h}{b}\int_0^b x^2\, dx$$

$$\frac{h}{b}\left[\frac{x^3}{3}\right]\Bigg|_0^b$$

$$\int \tilde{x}\, dA = \frac{hb^2}{3}$$

$$\bar{X} = \frac{\int \tilde{x}\, dA}{\int dA}$$

$$\bar{X} = \frac{\frac{hb^2}{3}}{\frac{bh}{2}}$$

$$\bar{X} = \frac{2\cancel{h}b^{\cancel{2}}}{3\cancel{b}\cancel{h}}$$

$$\boxed{\bar{X} = \frac{2}{3}b}$$

$\bar{y}$

$\int \tilde{y}\, dA$

$\int \left(\frac{h}{2b}x\right)\left(\frac{h}{b}x\right) dx$

$\frac{h^2}{2b^2}\int_0^b x^2\, dx$

$\frac{h^2}{2b^2}\left[\frac{x^3}{3}\right]_0^b$

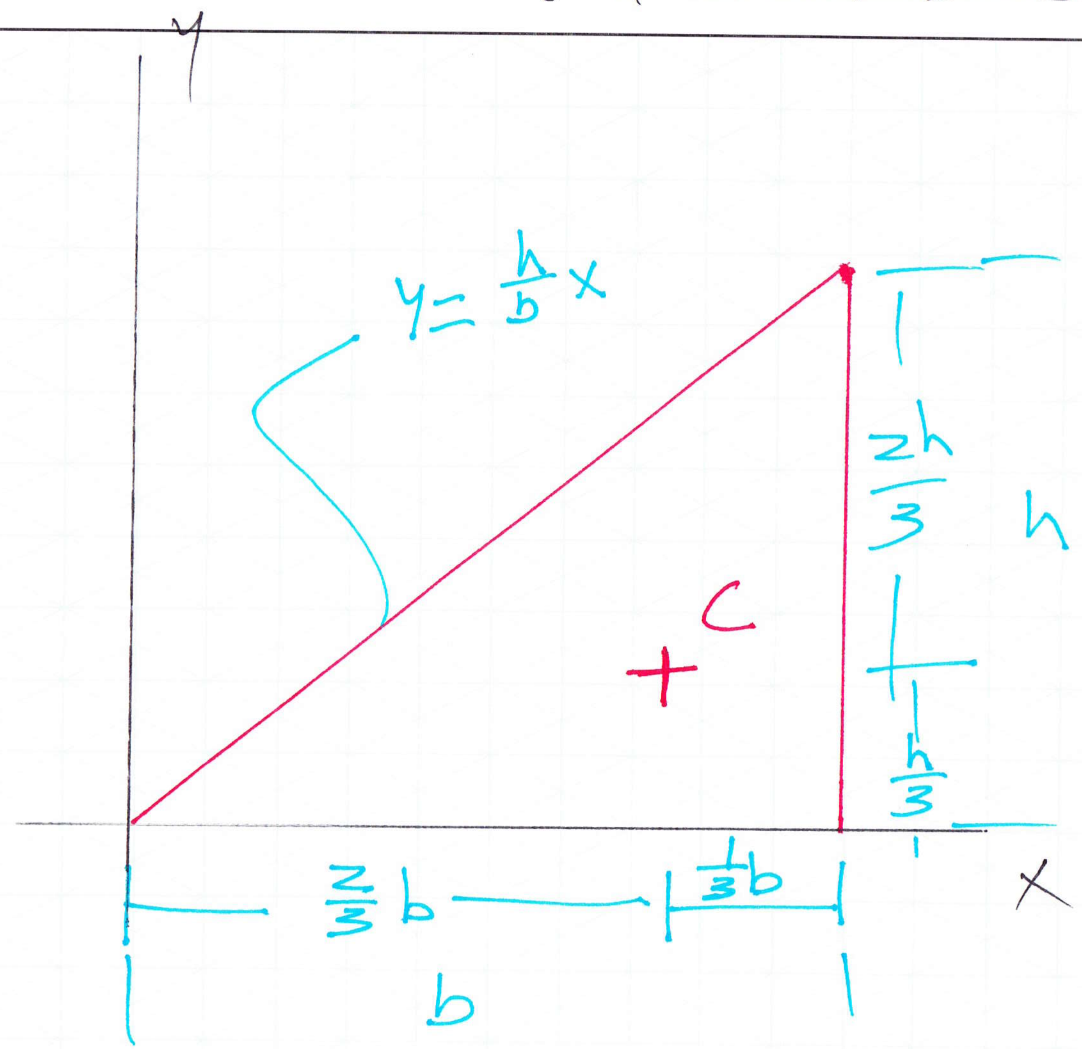
$\int \tilde{y}\, dA = \frac{h^2 b}{6}$

$\bar{y} = \frac{\int \tilde{y}\, dA}{\int dA}$

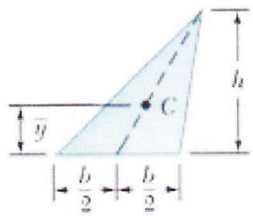
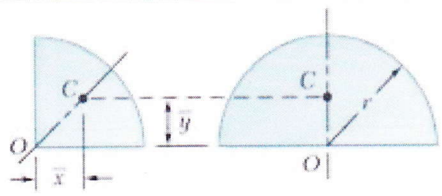
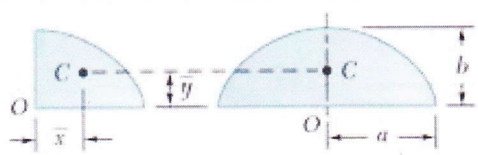
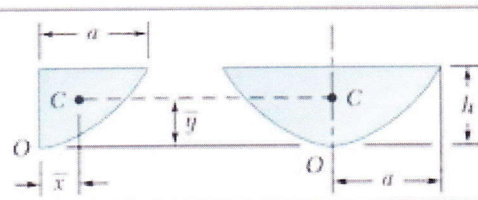
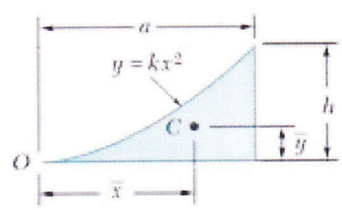
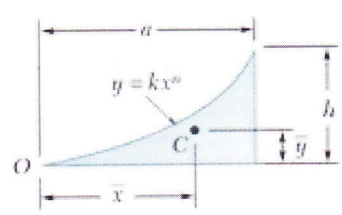
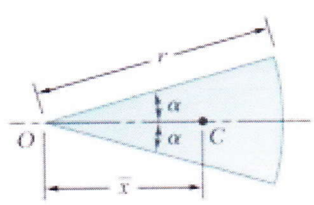
$\bar{y} = \frac{\frac{h^2 b}{6}}{\frac{bh}{2}}$

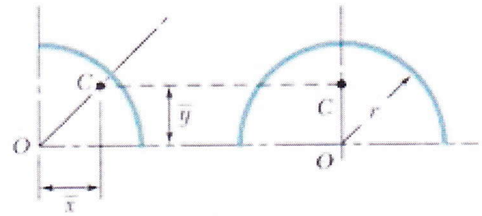
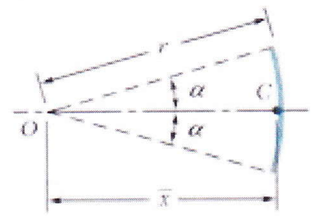
$\bar{y} = \frac{2h^{\cancel{2}}\cancel{b}}{6\cancel{b}\cancel{h}}$

$\boxed{\bar{y} = \frac{1}{3}h}$

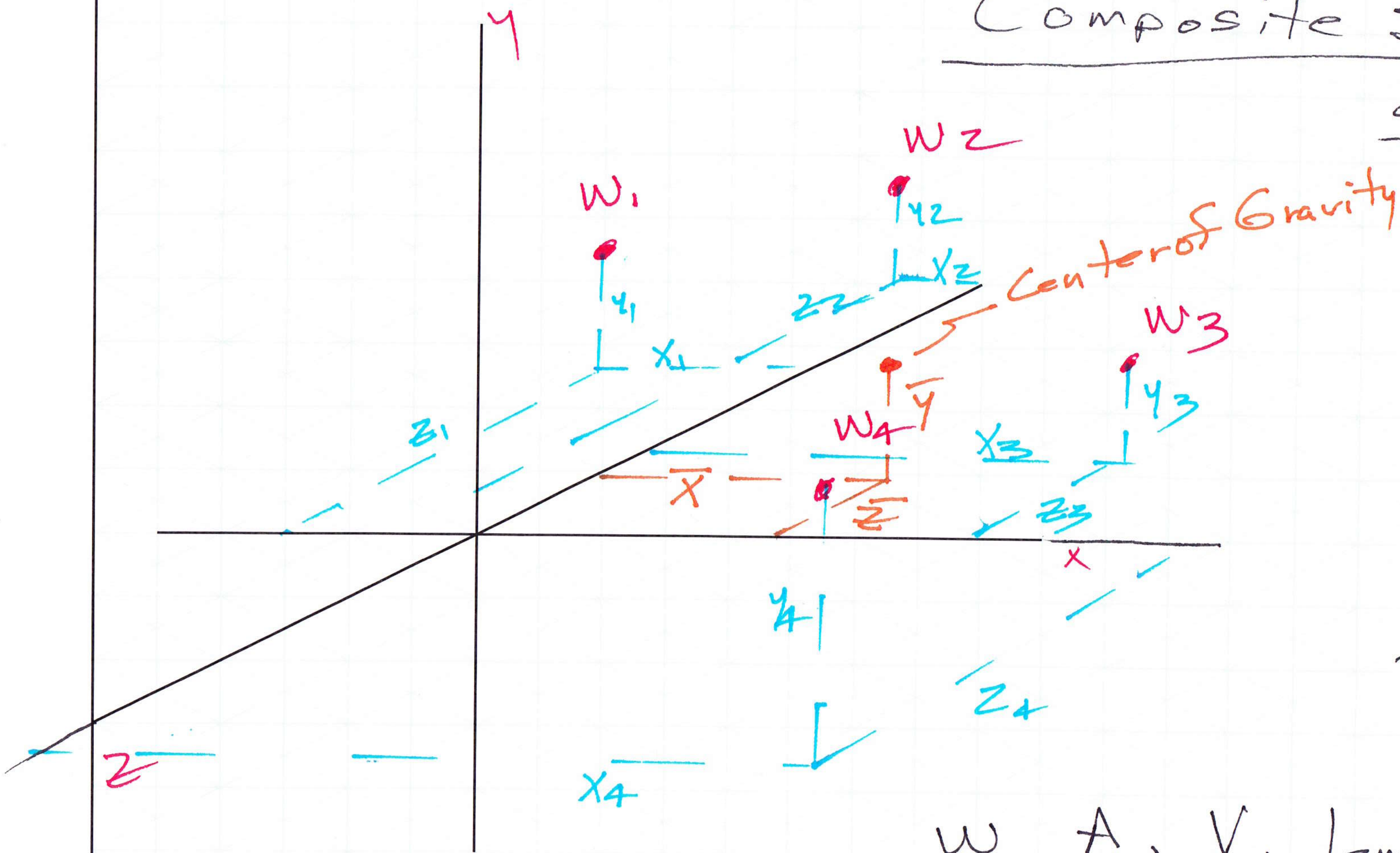


$$A = \frac{bh}{2}$$

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

Shape		$\overline{x}$	$\overline{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r \sin \alpha}{\alpha}$	0	$2\alpha r$

Composite Bodies – Center of Gravity of Discrete Weights



$$\bar{X} = \frac{\Sigma x_i w_i}{\Sigma w_i}$$

$$\bar{Y} = \frac{\Sigma y_i w_i}{\Sigma w_i}$$

$$\bar{Z} = \frac{\Sigma z_i w_i}{\Sigma w_i}$$

W, A, V, lengths, mass

Statics

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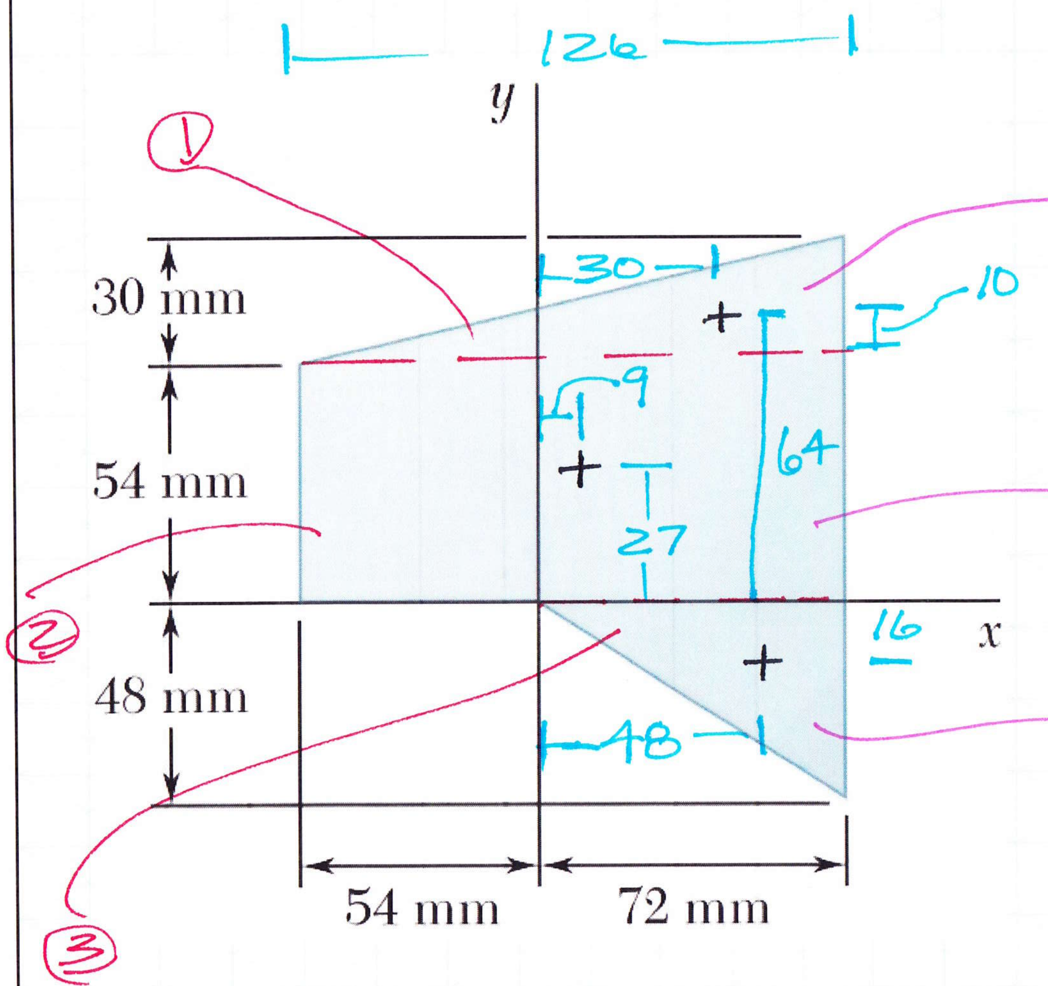

Locate the centroid of the shaded area shown. Use the method of composite areas.

$A_1 = \frac{1}{2}(126)(30) = 1890 \text{ mm}^2$

$A_2 = (126)(54) = 6{,}804 \text{ mm}^2$

$A_3 = \frac{1}{2}(72)(48) = 1{,}728 \text{ mm}^2$

Area	A_i	X_i	$A_i X_i$	Y_i	$A_i Y_i$
1	1,890	30	56,700	64	120,960
2	6,804	9	61,236	27	183,708
3	1,728	48	82,944	−16	−27,648
	10,422 mm²		200,880 mm³		277,020 mm³

$$\bar{X} = \frac{\Sigma A_i X_i}{\Sigma A_i} = \frac{200,880}{10,422} = \underline{19.27\text{ mm}}$$

$$\bar{Y} = \frac{\Sigma A_i Y_i}{\Sigma A_i} = \frac{277,020}{10,422} = \underline{26.58\text{ mm}}$$