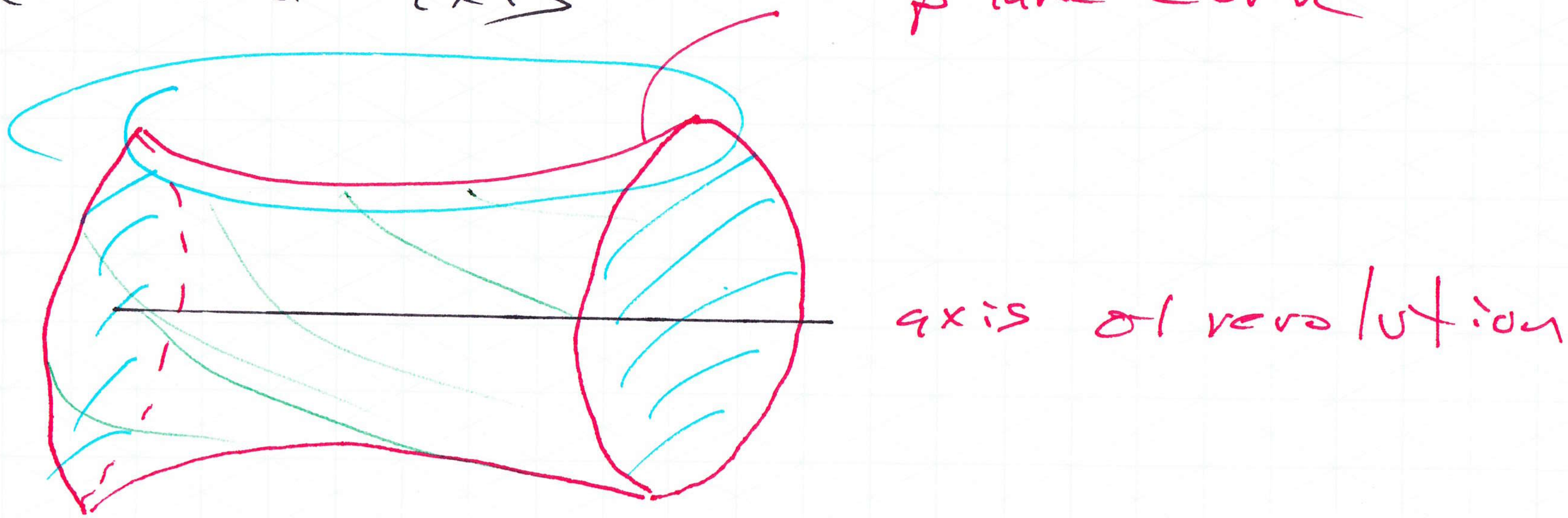
Theorems of Pappus-Guldinus

Solids and Surfaces of Revolution

Surface of revolution — a surface which can be generated by rotating a plane curve about a fixed axis



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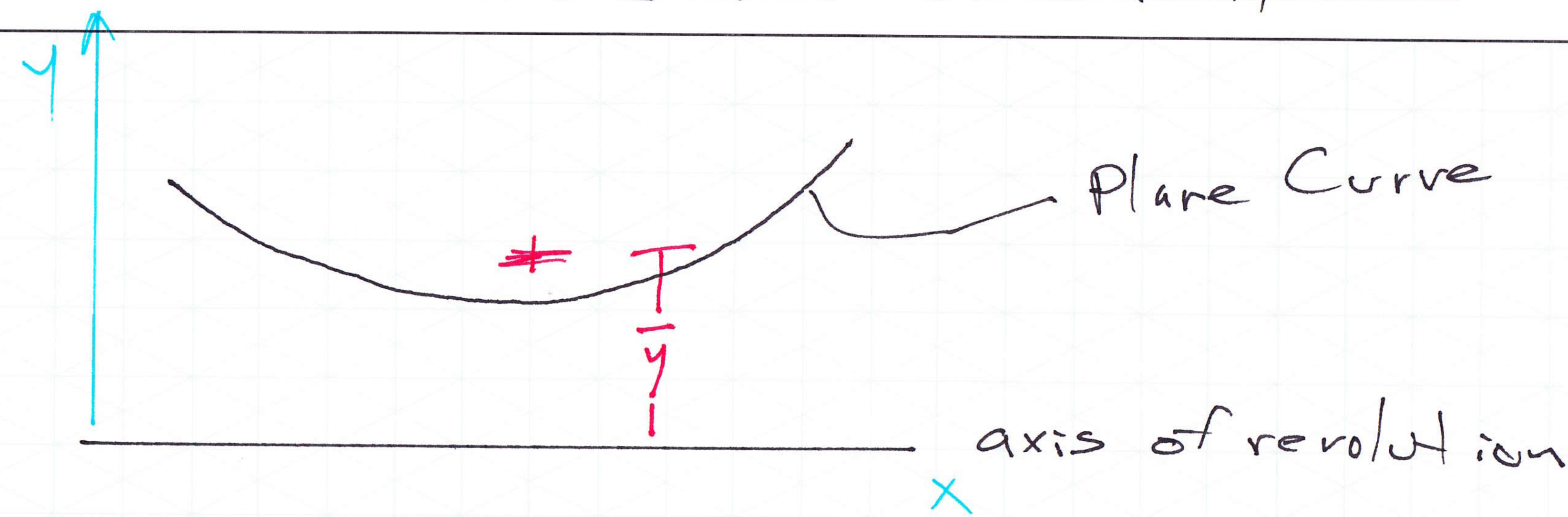

Theorem 1. – The area of a surface of revolution is equal to the product of the length of the generating curve and the distance traveled by the centroid of the generating curve as the surface is generated.

radians

$$A = \underbrace{\theta \, \bar{y}}_{\text{distance traveled by the centroid}} L$$

$\bar{y}$: distance from the centroid to the generating axis.

If it is a complete revolution

$$A = 2\pi \bar{y} L$$

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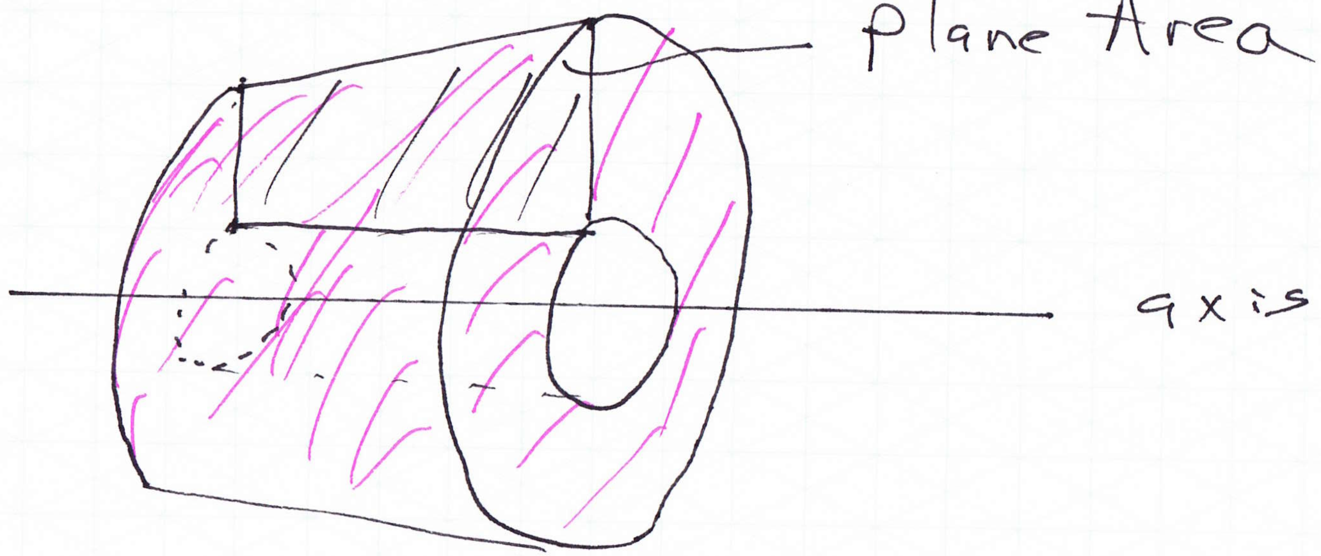
<u>Body of revolution</u> – generated by rotating a plane area about a fixed axis.



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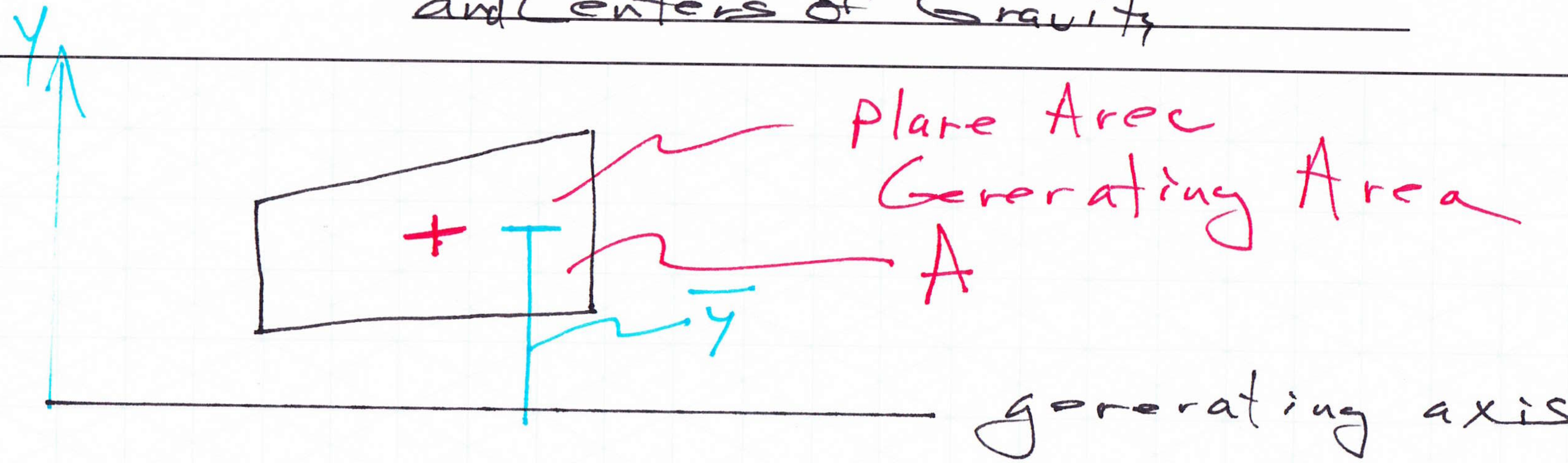

Theorem 2 — The volume of a body of revolution is equal to the product of the generating area and the distance traveled by the centroid of the area while the body is being generated.

$$V = \theta \bar{y} A$$

radians

Area

distance traveled by the centroid of the area

Complete 2π revolution

$$V = 2\pi \bar{y} A$$

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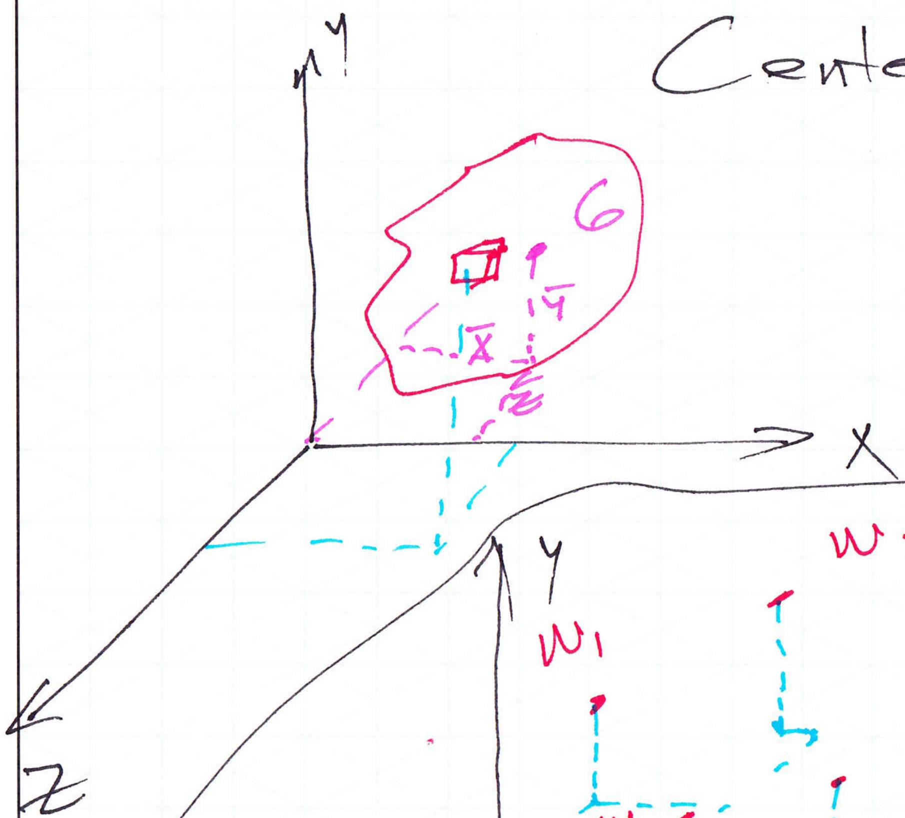
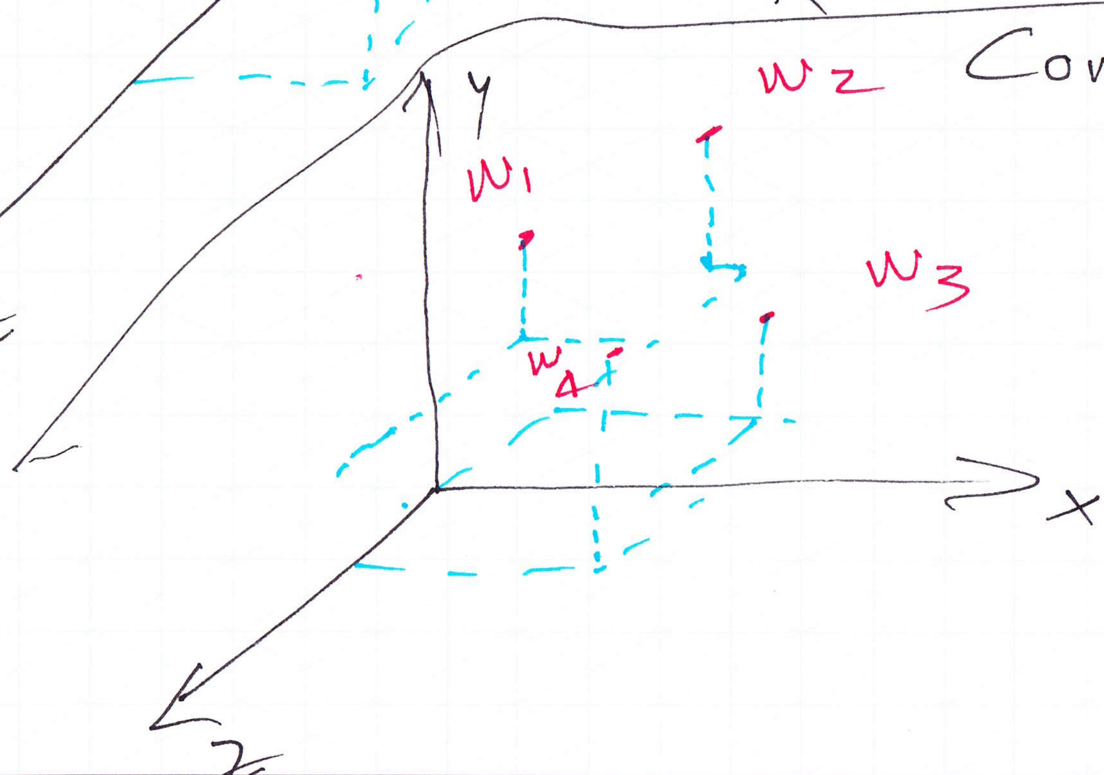
Center of Gravity, Centroid



$$\bar{X} = \frac{\int x\,dV}{V}, \qquad \bar{Y} = \frac{\int y\,dV}{V}$$

$$\bar{Z} = \frac{\int z\,dV}{V}$$

Composite ~~Body~~ Approach.



$$\bar{X} = \frac{\sum x_i w_i}{\sum w_i}$$

$$\bar{Y} = \frac{\sum y_i w_i}{\sum w_i}$$

$$\bar{Z} = \frac{\sum z_i w_i}{\sum w_i}$$

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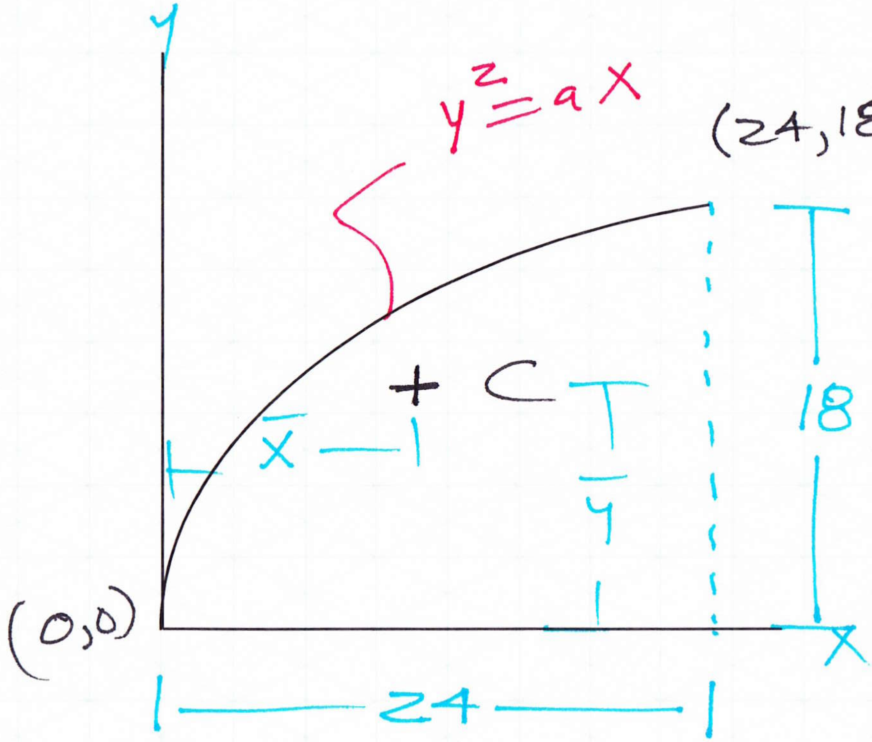

Locate the centroid for a slender Rod

First Thing solve for a

$$y^2 = a\,X$$

$$18^2 = a\,24$$

$$\underline{a = 13.5}$$

$$y^2 = 13.5\,X$$

$$y = \sqrt{13.5\,X}$$

or

$$X = \frac{y^2}{13.5}$$

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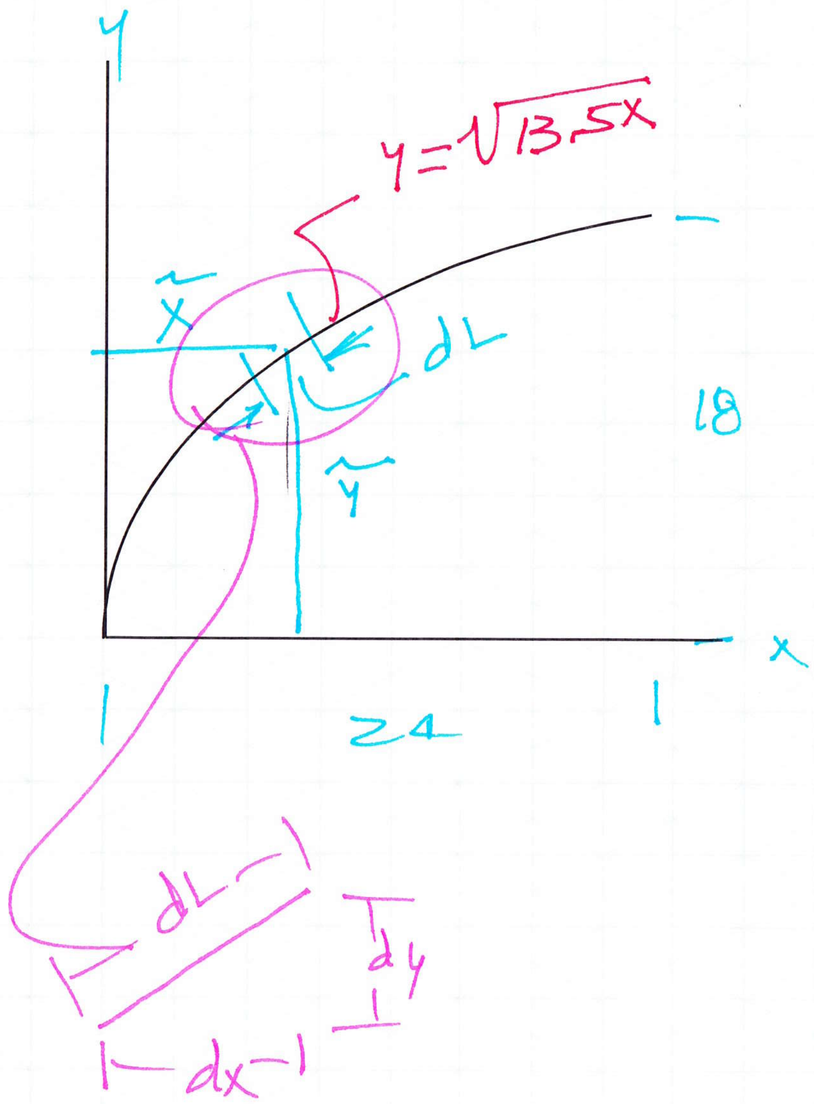

$\bar{x}$ and $\bar{y}$

tilde

$$\bar{x} = \frac{\int \tilde{x}\, dL}{\int dL} \qquad \bar{y} = \frac{\int \tilde{y}\, dL}{\int dL}$$

$$dL^2 = dx^2 + dy^2$$

$$dL = \sqrt{dx^2 + dy^2}$$

$$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

derivative of the function w.r.t. to x

$$y = \sqrt{13.5x}$$

$$\frac{dy}{dx} = \sqrt{\frac{13.5}{4x}}$$

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$$L = \int dL$$

$$L = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_0^{24} \sqrt{1 + \frac{13.5}{4x}}\, dx$$

$$\underline{L = 31.40}$$

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$$M_x = \int y \, dL$$

$$M_x = \int_0^{24} \left(\sqrt{13.5x}\right)\left(\sqrt{1 + \frac{13.5}{4x}} \, dx\right)$$

$$M_x = 335.65$$

$$\bar{y} = \frac{\int \tilde{y} \, dL}{\int dL} = \frac{M_x}{L} = \frac{335.65}{31.40}$$

$$\boxed{\bar{y} = 10.69}$$

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$$M_y = \int \tilde{x}\, dL$$

$$M_y = \int_0^{24} x \sqrt{1+\left(\frac{13.5}{4x}\right)}\, dx$$

$$\underline{M_y = 324.35}$$

$$\bar{X} = \frac{\int \tilde{x}\, dL}{\int dL} = \frac{M_y}{L} = \frac{324.35}{31.40}$$

$$\boxed{\bar{X} = 10.33}$$

$$\boxed{\bar{Y} = 10.69}$$

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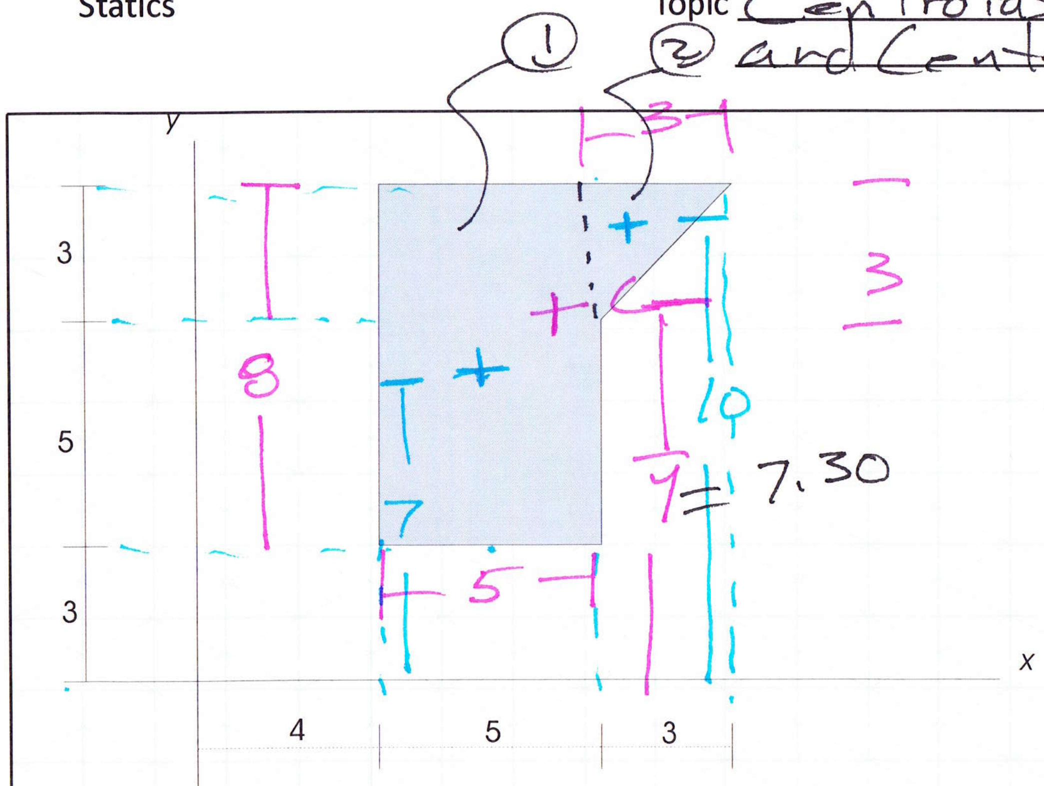

Volume generated by revolving the plane area, 360° about the x-axis. 2π radians

$A_1 = (5)(8) = 40$, $A_2 = \frac{1}{2}(3)(3) = 4.5$

$A_{Total} = A_1 + A_2 = 40 + 4.5 = \underline{44.5}$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_{total}}$$

$$\bar{y} = \frac{(40)(7) + 4.5(10)}{44.5}$$

$$\underline{\bar{y} = 7.30}$$

$V = 2\pi(7.30)(44.5)$

$V = 2041$ in^3 ft^3 m^3

rotating the area 360° about the x axis

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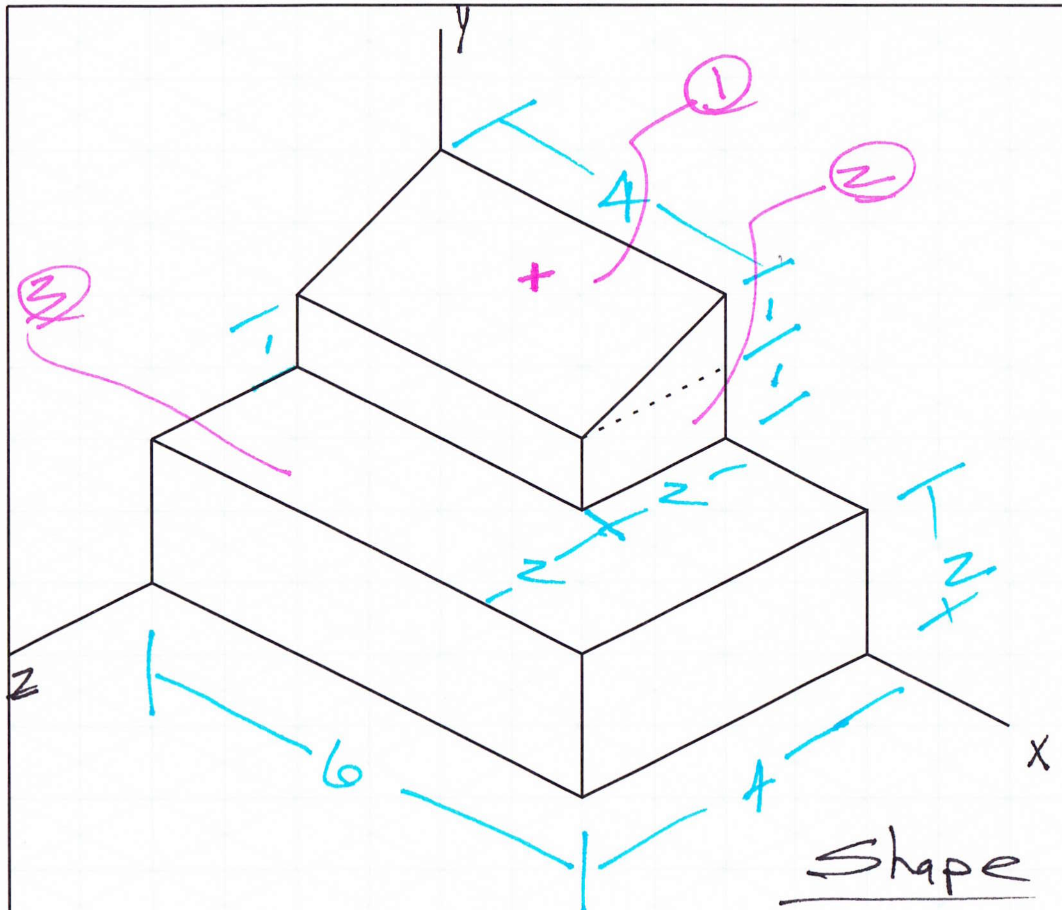
Centroid of a 3-D Body



$$V_1 = \tfrac{1}{2}(2)(1)(4) = 4$$

$$V_2 = (4)(2)(1) = 8$$

$$V_3 = (6)(4)(2) = 48$$

$$V_{Tot} = V_1 + V_2 + V_3$$

$$V_{Tot} = 4 + 8 + 48 = \underline{60}$$

Shape	$\tilde{x}_i$	$\tilde{y}_i$	$\tilde{z}_i$
1	2	$3\frac{1}{3}$	$\frac{2}{3}$
2	2	$2\frac{1}{2}$	1
3	3	1	2

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Shape	V_i	$\tilde{x}_i$	$\tilde{x}_i V_i$	$\tilde{y}_i$	$\tilde{y}_i V_i$	$\tilde{z}_i$	$\tilde{z}_i V_i$
1	4	2	8	3.333	13.33	.667	2.67
2	8	2	16	2.50	20	1	8
3	48	3	144	1	48	2	96
	60		168		81.33		106.67

$$\bar{x} = \frac{\Sigma \tilde{x}_i V_i}{\Sigma V_i} \qquad \bar{y} = \frac{\Sigma \tilde{y}_i V_i}{\Sigma V_i} \qquad \bar{z} = \frac{\Sigma \tilde{z}_i V_i}{\Sigma V_i}$$

$$\bar{x} = \frac{168}{60} \qquad \bar{y} = \frac{81.33}{60} \qquad \bar{z} = \frac{106.67}{60}$$

$$\underline{\bar{x} = 2.80} \qquad \underline{\bar{y} = 1.36} \qquad \underline{\bar{z} = 1.78}$$

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