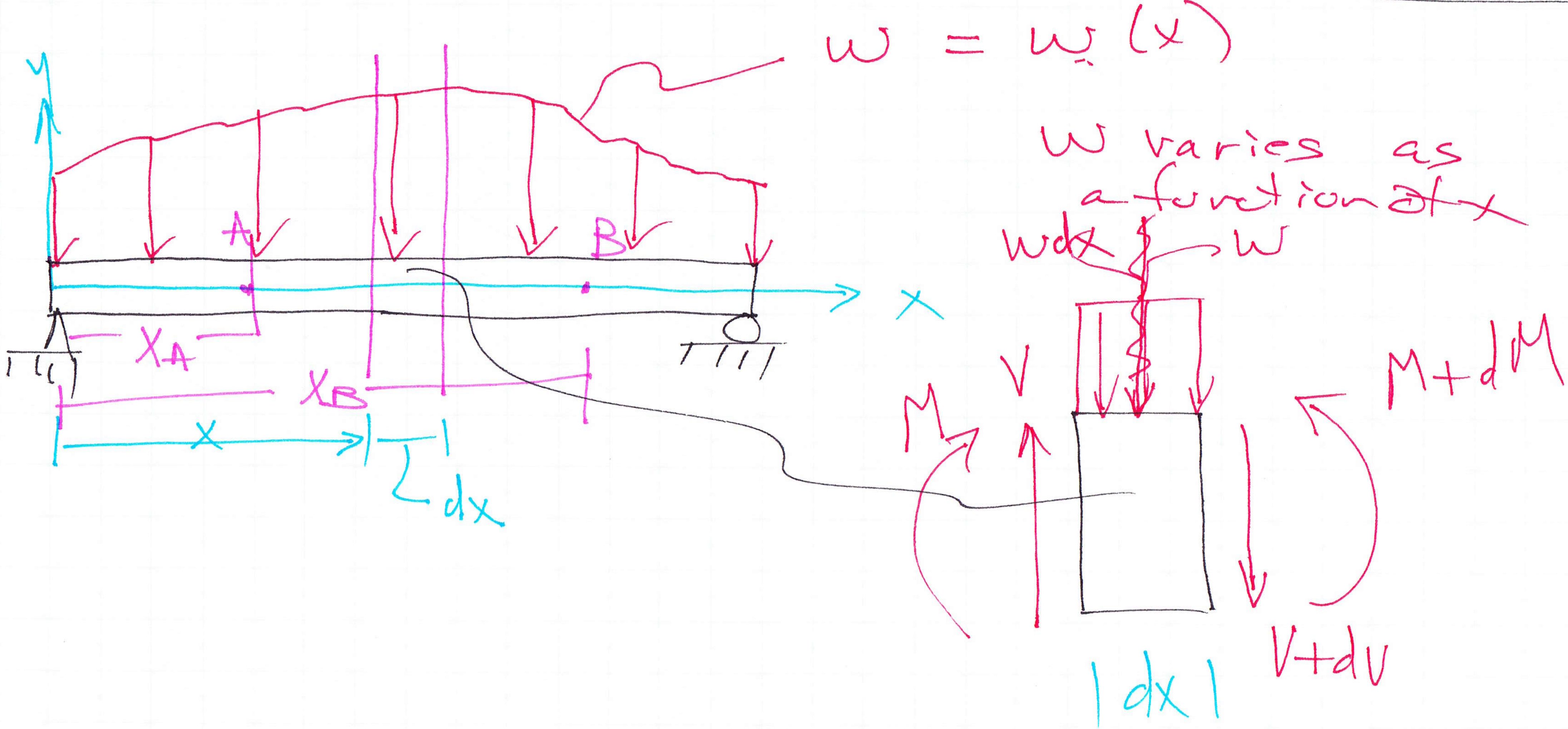
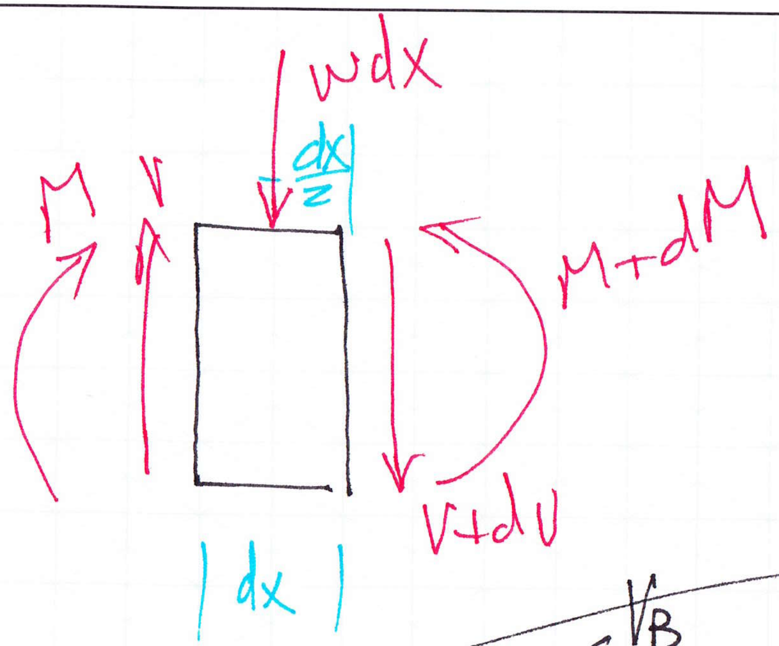
Relationships between Load, shear, and Moment





$$+\uparrow \sum F_y = 0$$

$$V - w\,dx - (V + dV) = 0$$

$$-dV - w\,dx = 0$$

$$dV = -w\,dx$$

$$\frac{dV}{dx} = -w$$

$\frac{dV}{dx}$: slope of the shear diagram

w: load intensity + downward

$$\int_{V_A}^{V_B} dV = -\int_{X_A}^{X_B} w\,dx$$

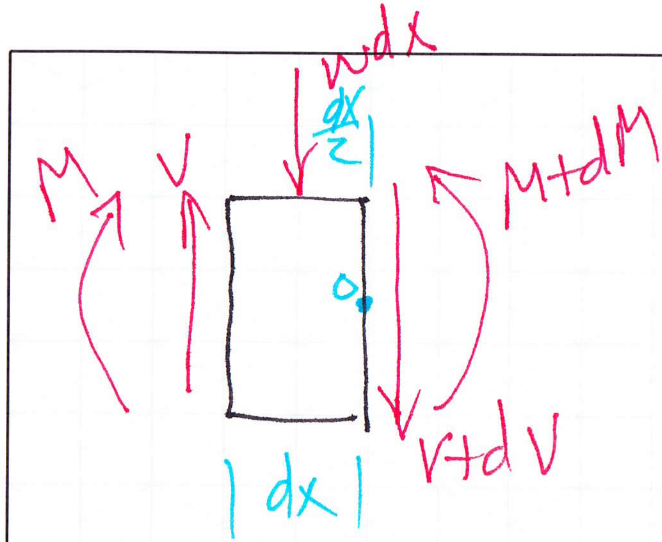
$\int w\,dx$: area under the load diagram between A and B

$$\Delta V_{AB} = V_B - V_A = -\int_{X_A}^{X_B} w\,dx$$

ΔV_{AB}: change in shear between points A and B.

Four Rules for Shear and Moment Diagrams

1. The slope of the shear diagram at any point is equal to $\ominus$ the load intensity at the same point.
2. The change in shear between two points on the shear diagram is equal to $\ominus$ the area under the loading diagram between the same two points.
3. The slope of the moment diagram at any point is equal to the shear at the same point.
4. The change in moment between two points on the moment diagram is equal to the area under the shear diagram between the same two points.



$$\circlearrowleft + \sum M_{\text{point } O} = 0$$

$$-M - V\,dx + w\,dx\,\frac{dx}{2} + M + dM = 0$$

$$dM = V\,dx - \frac{w}{2}\,dx^2 \rightarrow 0$$

$$dM = V\,dx$$

$$\int_{M_A}^{M_B} dM = \int_{X_A}^{X_B} V\,dx$$

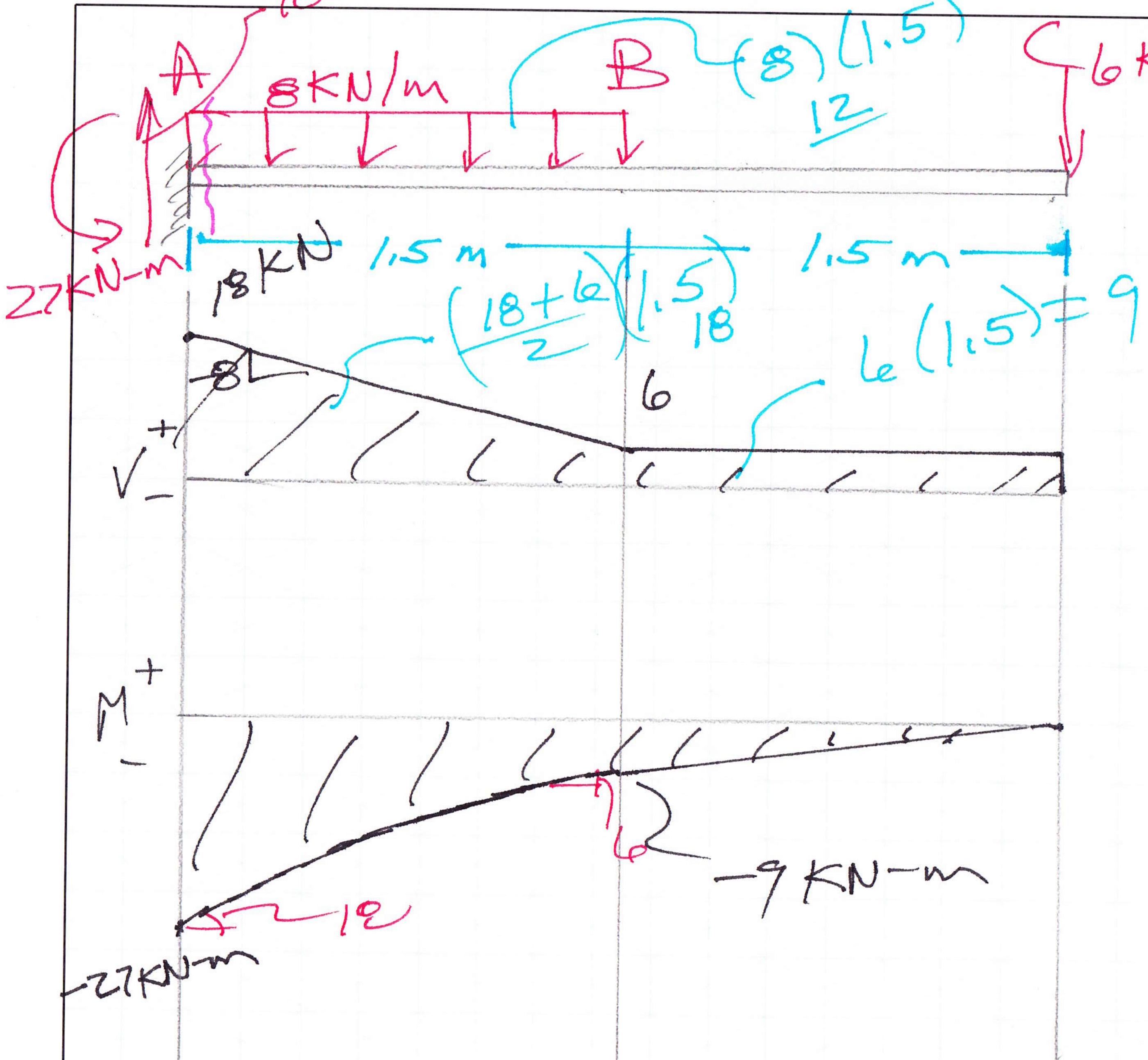
$$V = \frac{dM}{dx}$$

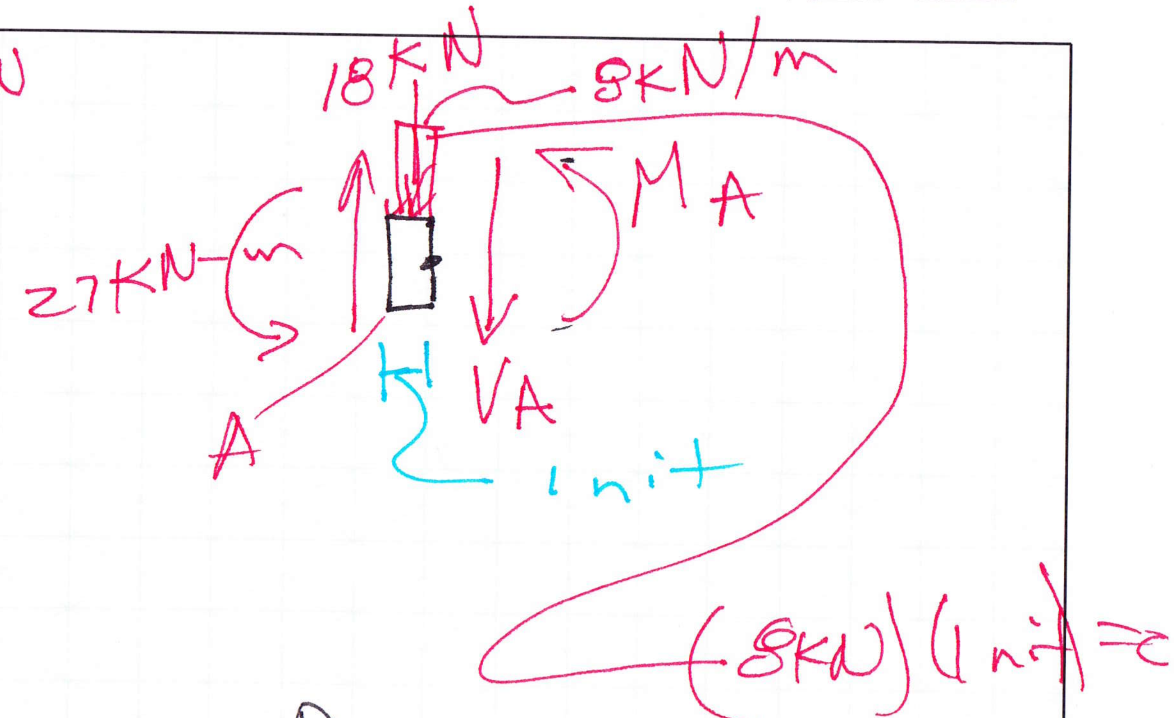
shear — 1st derivative of the moment

$$\Delta M_{AB} = M_B - M_A = \int_{X_A}^{X_B} V\,dx$$

change in moment between A and B

Area under the shear diagram between A and B





$\uparrow \Sigma F_y = 0$

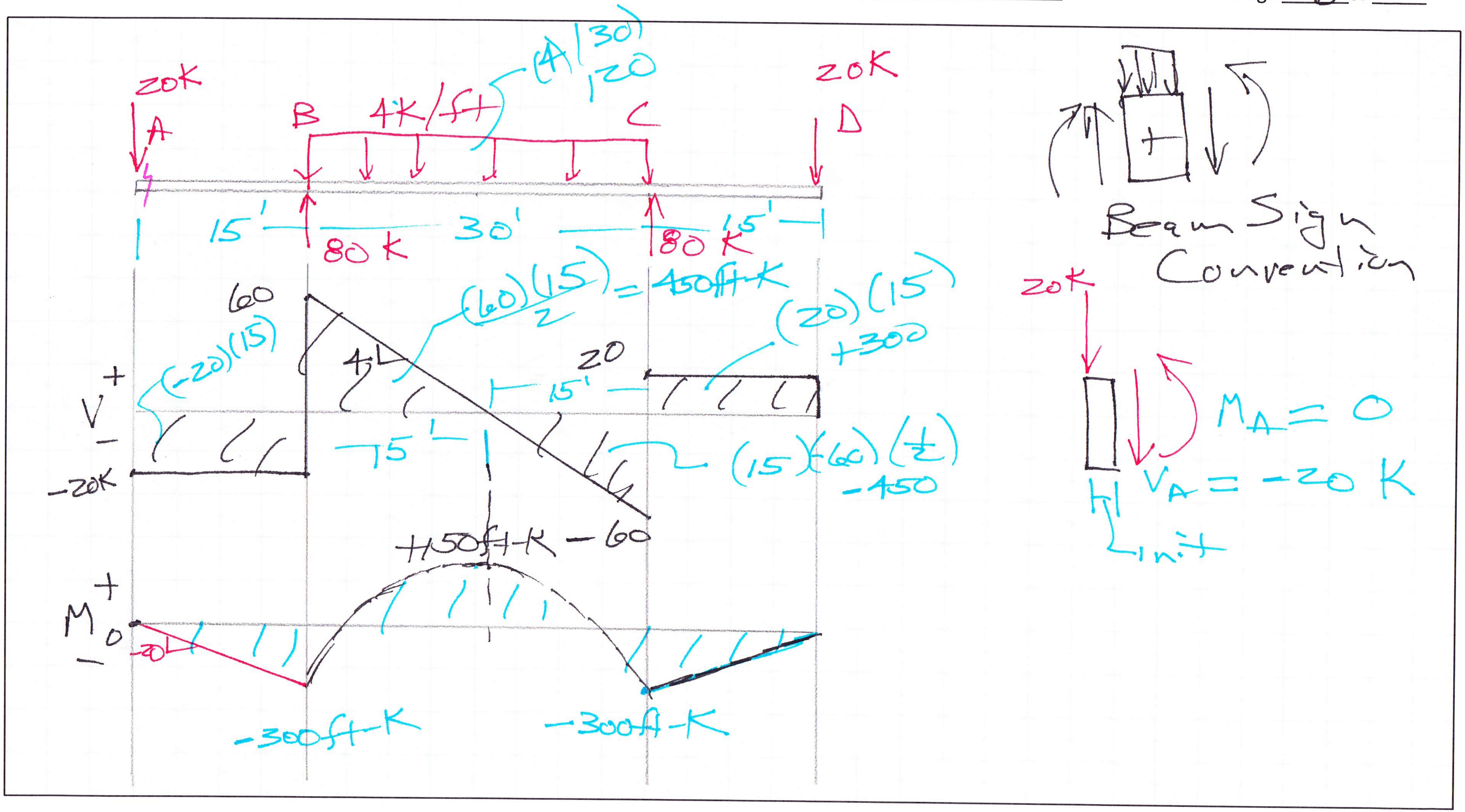
$18 - V_A = 0$

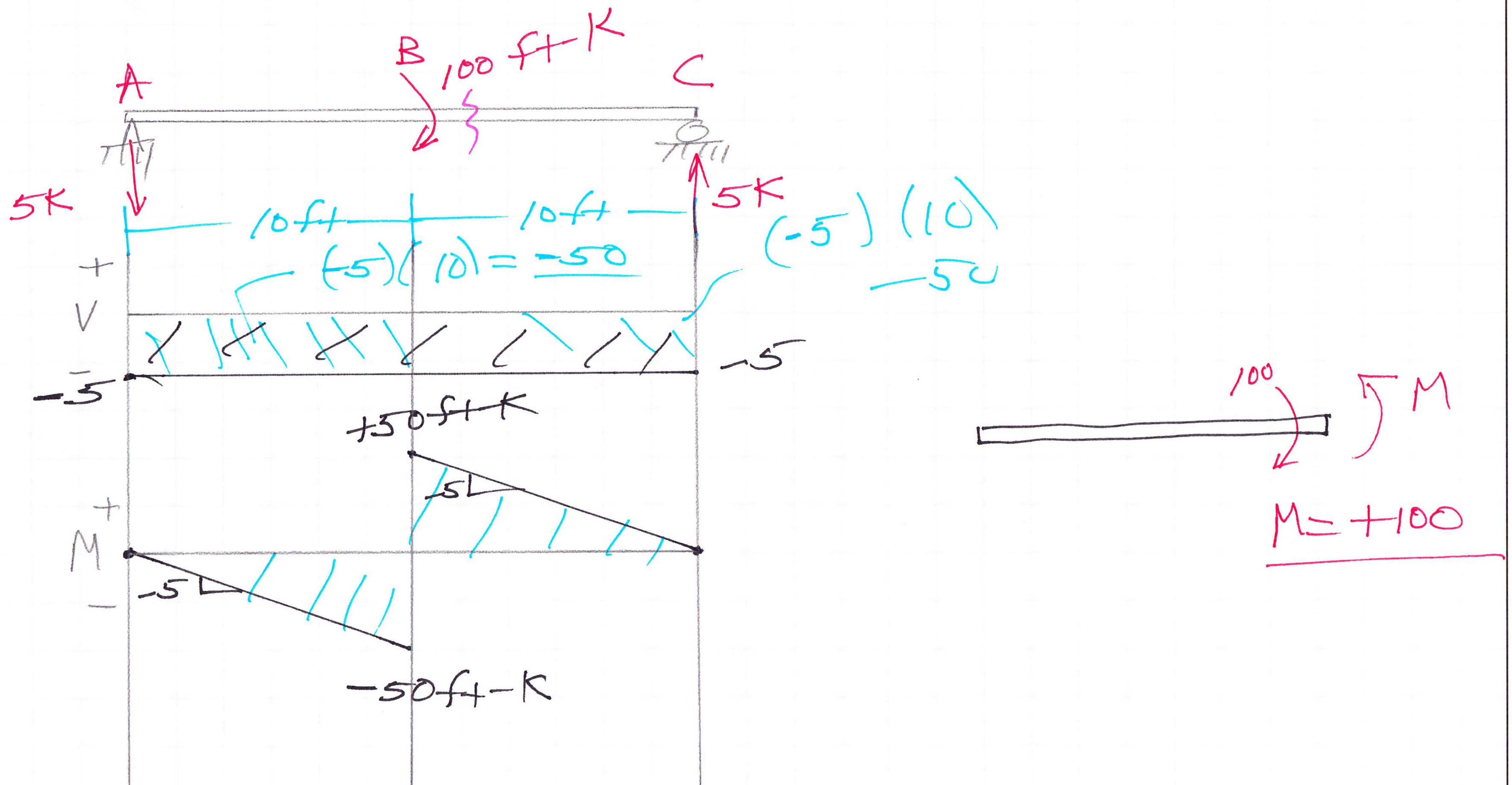
$V_A = 18$ KN

$\circlearrowleft + \Sigma M_{cut}$

$27 + M_A = 0$

$M_A = -27$ KN


20K
A
B
4K/ft
(4)(30)
120
C
20K
D
15'
30'
15'
80 K
80 K
60
(60)(15)/2 = 450ft-K
(-20)(15)
4
20
(20)(15)
+300
V
+
-
15'
-20K
7.5'
(15)(60)(1/2)
-450
+450ft-K
-60
M
+
0
-
-20
-300ft-K
-300ft-K
Beam Sign Convention
20K
$M_A = 0$
$V_A = -20K$
Limit


A
B
C
100 ft-K
5K
5K
10 ft
10 ft
(-5)(10) = -50
(-5)(10)
-50
+
V
-
-5
-5
+50 ft-K
-5
+
M
-
-5
-50 ft-K
100
M
M = +100

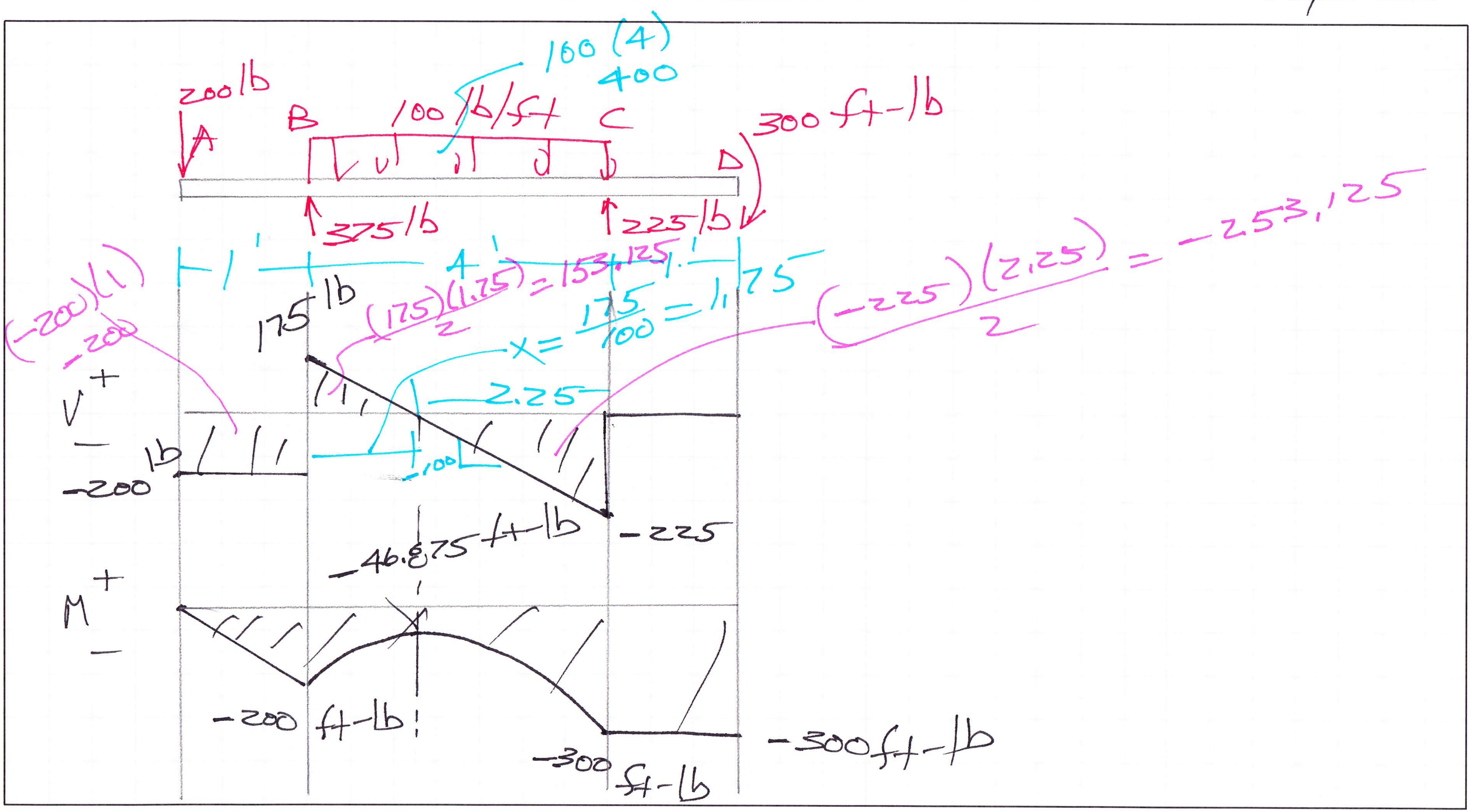
Always Go Left to Right

1) Downward external force moves the V diagram downward.

2) Upward external fone moves the V diagram upward

3) Clockwise external moment moves the M diagram upward.

4) Counter Clockwise external moment moves the M diagram downward.


200 lb
A
B
100 lb/ft
C
100 (4)
400
300 ft-lb
D
375 lb
225 lb
1'
4'
1'
(-200)(1)
-200
175 lb
(175)(1.75)/2 = 153.125
x = 175/100 = 1.75
2.25
(-225)(2.25)/2 = -253.125
V
-200 lb
100
-225
-46.875 ft-lb
M
-200 ft-lb!
-300 ft-lb
-300 ft-lb