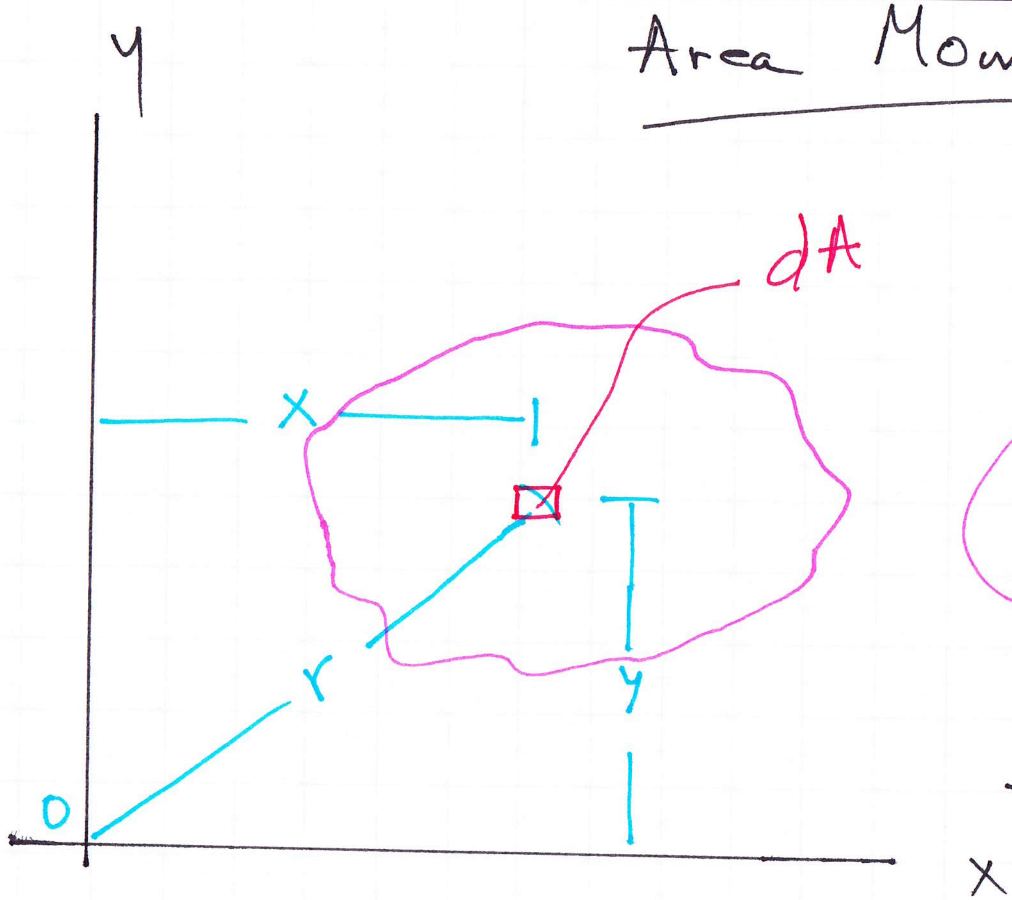
Area Moment of Inertia



$r^2 = x^2 + y^2$

By Definition

Bending Problems

$$I_x = \int_{Area} y^2 \, dA$$

$$I_y = \int_{Area} x^2 \, dA$$

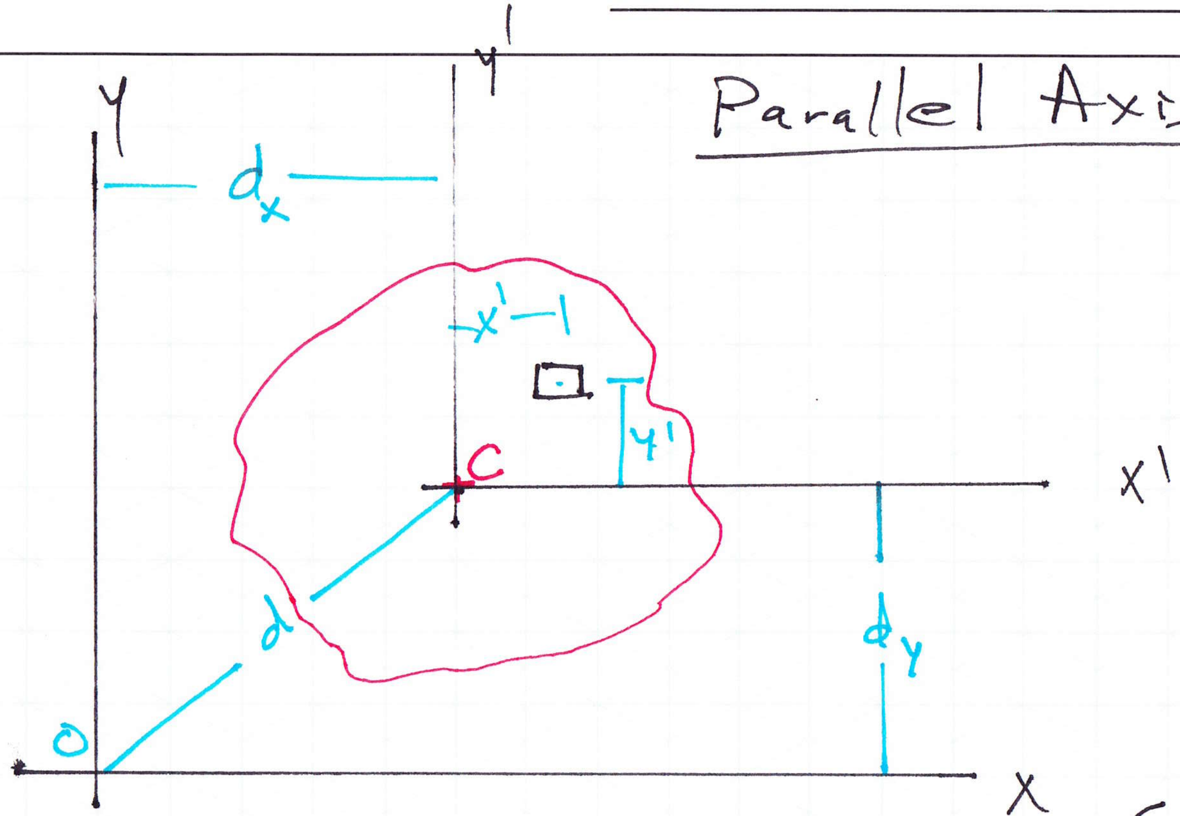
$$I_z = J_o = \int_{Area} r^2 \, dA$$

$$J_o = I_x + I_y$$

Polar Moment of Inertia

Torsion Problems

Parallel Axis Theorem



x' and y' are centroidal axes parallel to x and y respectively

$$I_x = \int_{Area} (y' + d_y)^2 \, dA$$

$$I_x = \int y'^2 \, dA + 2 d_y \cancel{\int y' \, dA}^{\,0} + d_y^2 \int dA$$

$$\boxed{I_x = \bar{I}_{x'} + d_y^2 A}$$

⊥ distance between parallel axes

This absolutely, positively, must be a centroidal axis

$$I_y = \bar{I}_{y'} + d_x^2 A$$

in^4 ft^4

$$J_O = \bar{J}_C + A d^2$$

m^4 mm^4

$\cancel{cm^4}$

Radius of Gyration

$$K_x = \sqrt{\frac{I_x}{A}} \qquad I_x = K_x^2 A$$

in

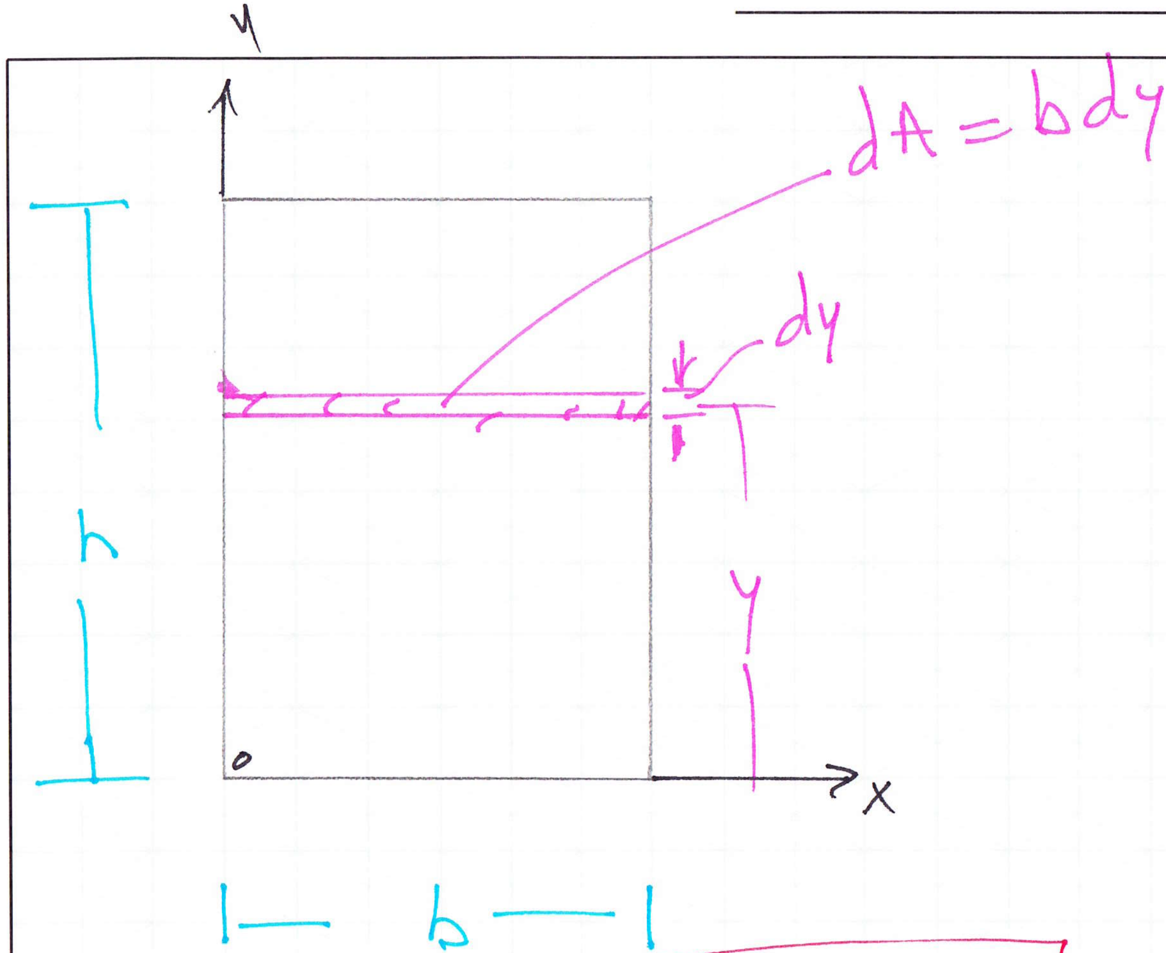
ft

$$K_y = \sqrt{\frac{I_y}{A}} \qquad I_y = K_y^2 A$$

m

mm

$$K_O = \sqrt{\frac{J_O}{A}} \qquad J_O = K_O^2 A$$



$$\boxed{I_y = \frac{b^3 h}{3}}$$

I_x –

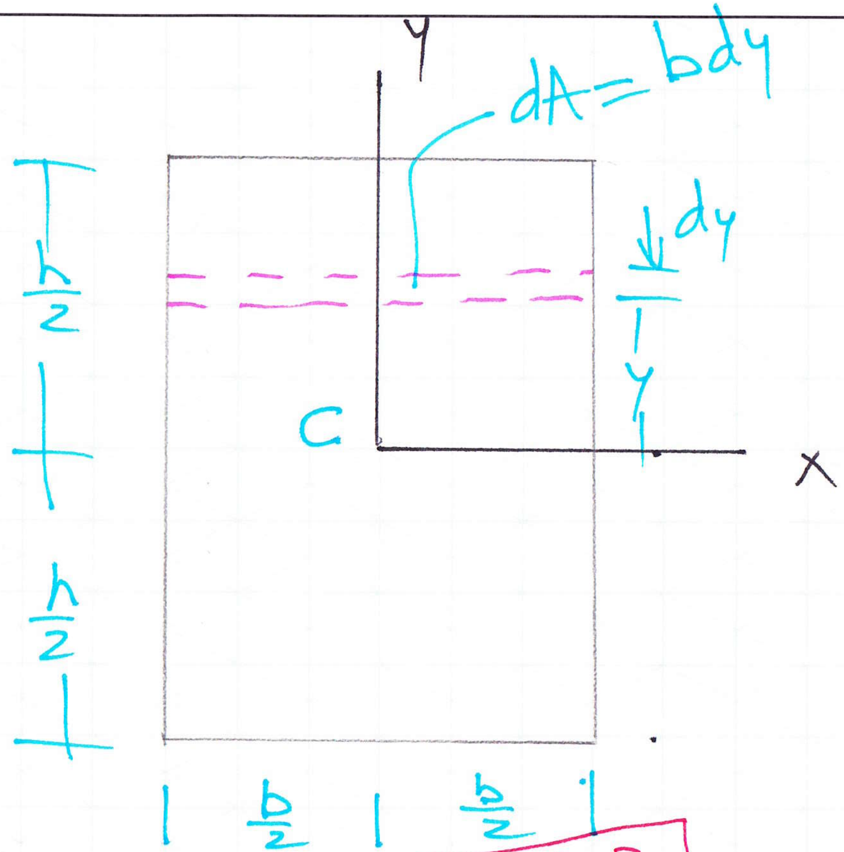
dA easiest if dA is parallel to the x-axis

$$I_x = \int y^2 \, dA$$

$$I_x = b \int_0^h y^2 \, dy$$

$$I_x = b \, \frac{1}{3} \, y^3 \Big|_0^h$$

$$\boxed{I_x = \frac{b h^3}{3}}$$



$$\boxed{I_y = \frac{hb^3}{12}}$$

both the x and y axes are centroidal

$$I_x = \int y^2 \, dA$$

$$I_x = b \int_{-\frac{h}{2}}^{+\frac{h}{2}} y^2 \, dA$$

$$I_x = b \frac{1}{3} y^3 \Big|_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$I_x = \frac{b}{3} \left[\frac{h^3}{8} - \left(-\frac{h^3}{8} \right) \right]$$

$$\boxed{\bar{I}_x = \frac{bh^3}{12}}$$

the moment of inertia of rectangular area about its own centroidal axis

Engineering Proof

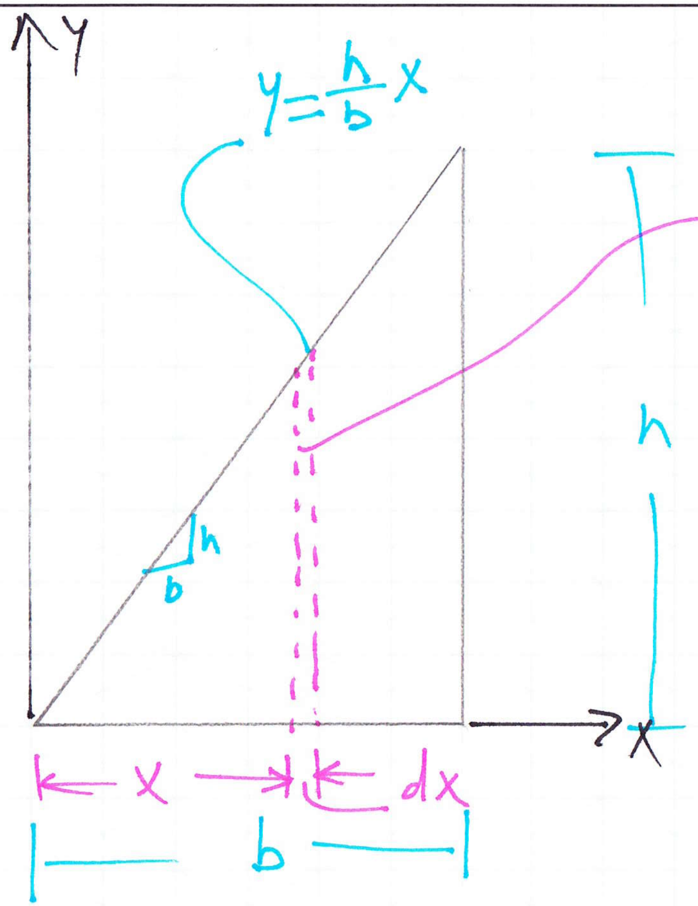
$$I_x = \bar{I}_x + A d_y^2$$

$$\frac{bh^3}{3} = \frac{bh^3}{12} + bh\left(\frac{h}{2}\right)^2$$

$$\frac{bh^3}{3} = \frac{bh^3}{12} + \frac{bh^3}{4}$$

$$\frac{bh^3}{3} = \frac{bh^3}{3}$$

Parallel Axis Theorem is valid at least for rectangles.



I_x

$$dI_x = \frac{1}{3}\left(\frac{h}{b}x\right)^3 dx$$

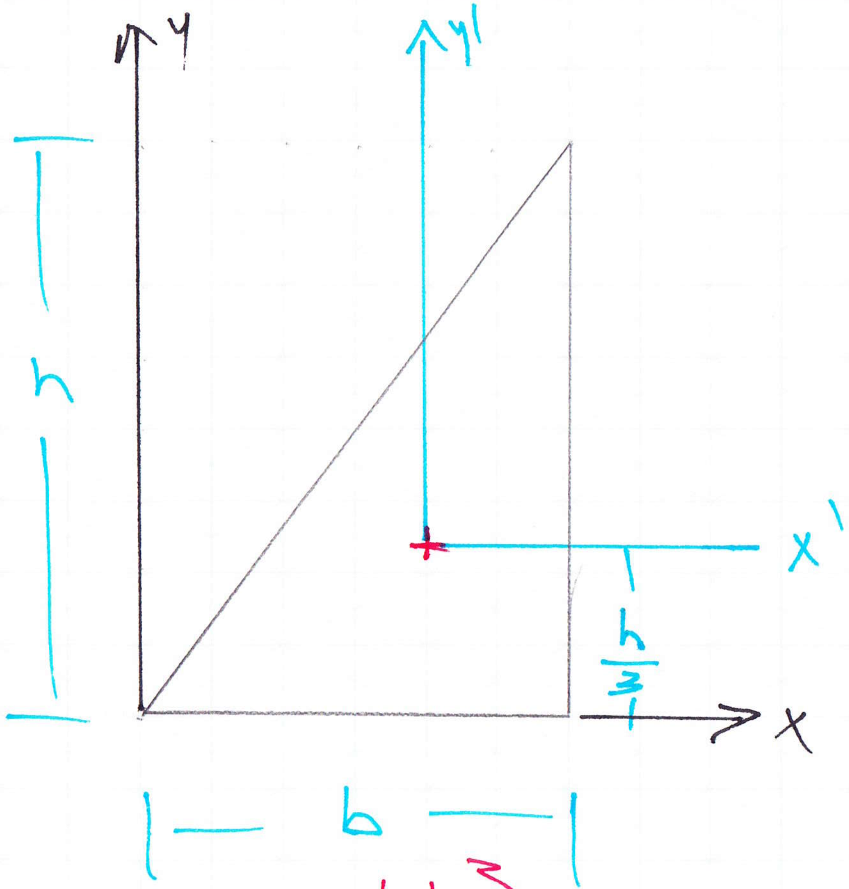
$$I_x = \int_0^b \frac{1}{3}\left(\frac{h}{b}x\right)^3 dx$$

$$I_x = \frac{h^3}{3b^3}\int_0^b x^3\, dx$$

$$I_x = \frac{h^3}{3b^3}\,\frac{1}{4}x^4\Big|_0^b$$

$$I_x = \frac{h^3}{12\cancel{b^3}}\,b^{\cancel{4}}$$

$$\boxed{I_x = \frac{bh^3}{12}}$$



$$I_x = \frac{bh^3}{12}$$

$$I_x = \bar{I}_{x'} + Ad^2$$

$$\frac{bh^3}{12} = \bar{I}_{x'} + \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2$$

$$\frac{bh^3}{12} = \bar{I}_{x'} + \frac{bh^3}{18}$$

$$\bar{I}_{x'} = \frac{bh^3}{12} - \frac{bh^3}{18}$$

$$\bar{I}_{x'} = \frac{3bh^3 - 2bh^3}{36}$$

$$\boxed{\bar{I}_{x'} = \frac{bh^3}{36}}$$