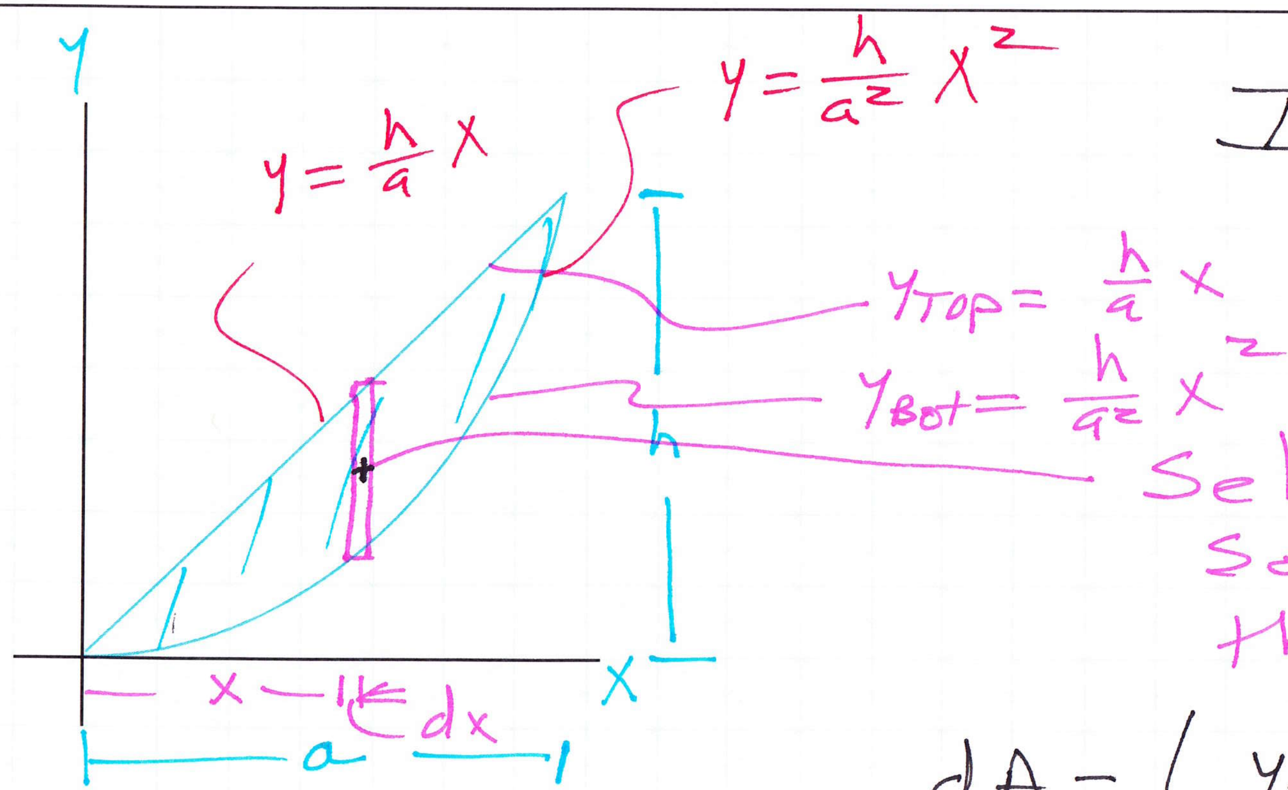


I_y — area moment of inertia about the y-axis

Selected the differential so that is parallel with the y-axis

$$dA = (y_{TOP} - y_{Bot})\,dx$$

$$dA = \left(\frac{h}{a}x - \frac{h}{a^2}x^2\right)dx$$

$$I_y = \int_{Area} x^2\,dA$$

(x: distance from the y-axis to the centroid of the differential element)

$$I_y = \int_0^a x^2 \left(\frac{h}{a}x - \frac{h}{a^2}x^2\right)dx$$

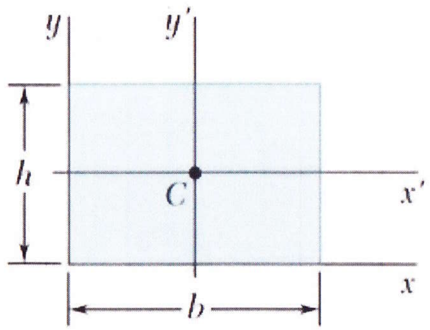
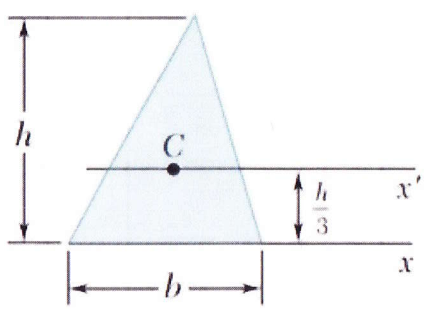
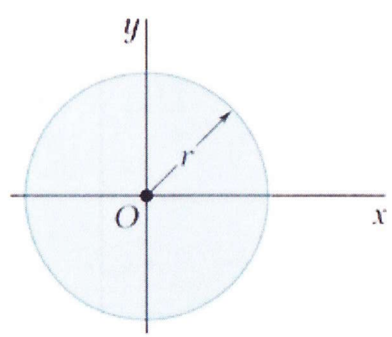
$$I_y = \frac{h}{a}\int_0^a x^3\,dx - \frac{h}{a^2}\int_0^a x^4\,dx$$

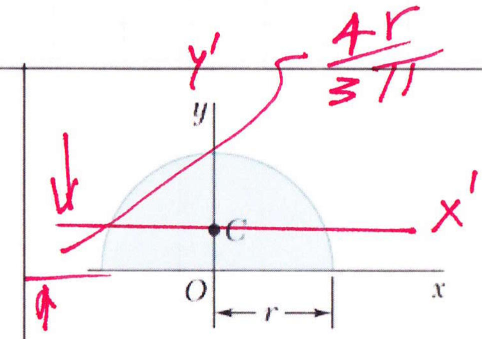
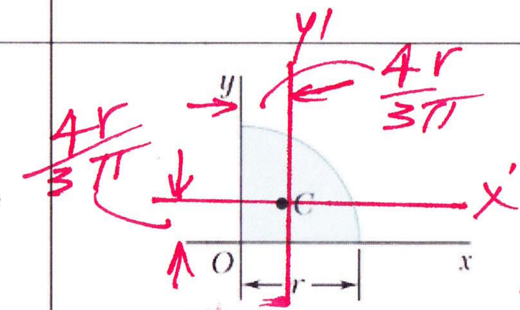
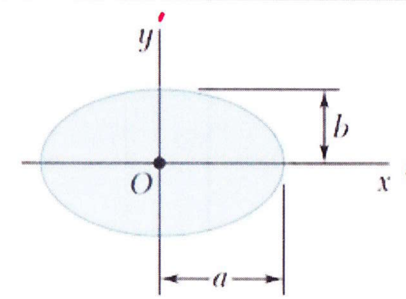
$$\frac{h}{a}\,\frac{x^4}{4}\Big|_0^a - \frac{h}{a^2}\,\frac{x^5}{5}\Big|_0^a$$

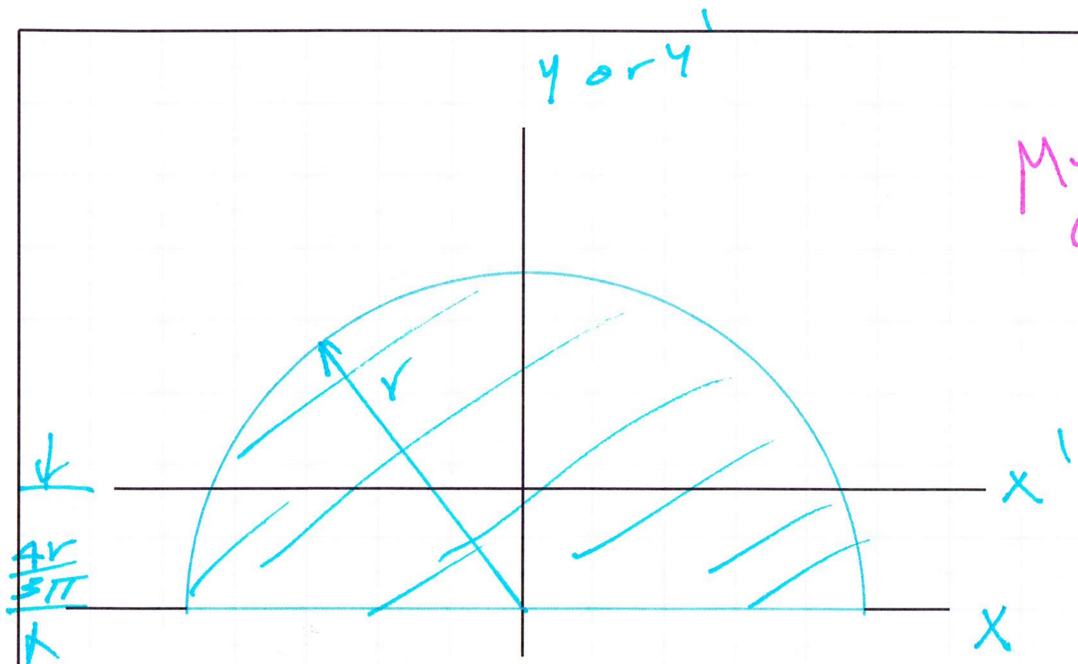
$$\frac{ha^3}{4} - \frac{ha^3}{5}$$

$$\frac{5ha^3 - 4ha^3}{20}$$

$$\boxed{I_y = \frac{ha^3}{20}}$$

Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$I_x = I_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$

Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4 = I_{y'}$ $J_O = \frac{1}{4}\pi r^4$ $I_{x'} = \frac{\pi r^4}{8} - \frac{8}{9}\frac{r^4}{\pi}$ $I_{x'} \simeq 0.1098\, r^4$
Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$ $I_{x'} = I_{y'} = \frac{\pi r^4}{16} - \frac{4}{9}\frac{r^4}{\pi}$ $I_{x'} = I_{y'} \simeq 0.05488\, r^4$
Ellipse $A = \pi ab$		$\bar{I}_{x'} = \frac{1}{4}\pi ab^3$ $\bar{I}_{y'} = \frac{1}{4}\pi a^3 b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$



$$I_x = \frac{1}{8}\pi r^4$$

From Table

Must be a centroidal axis

$$I_x = \bar{I}_{x'} + A d^2$$

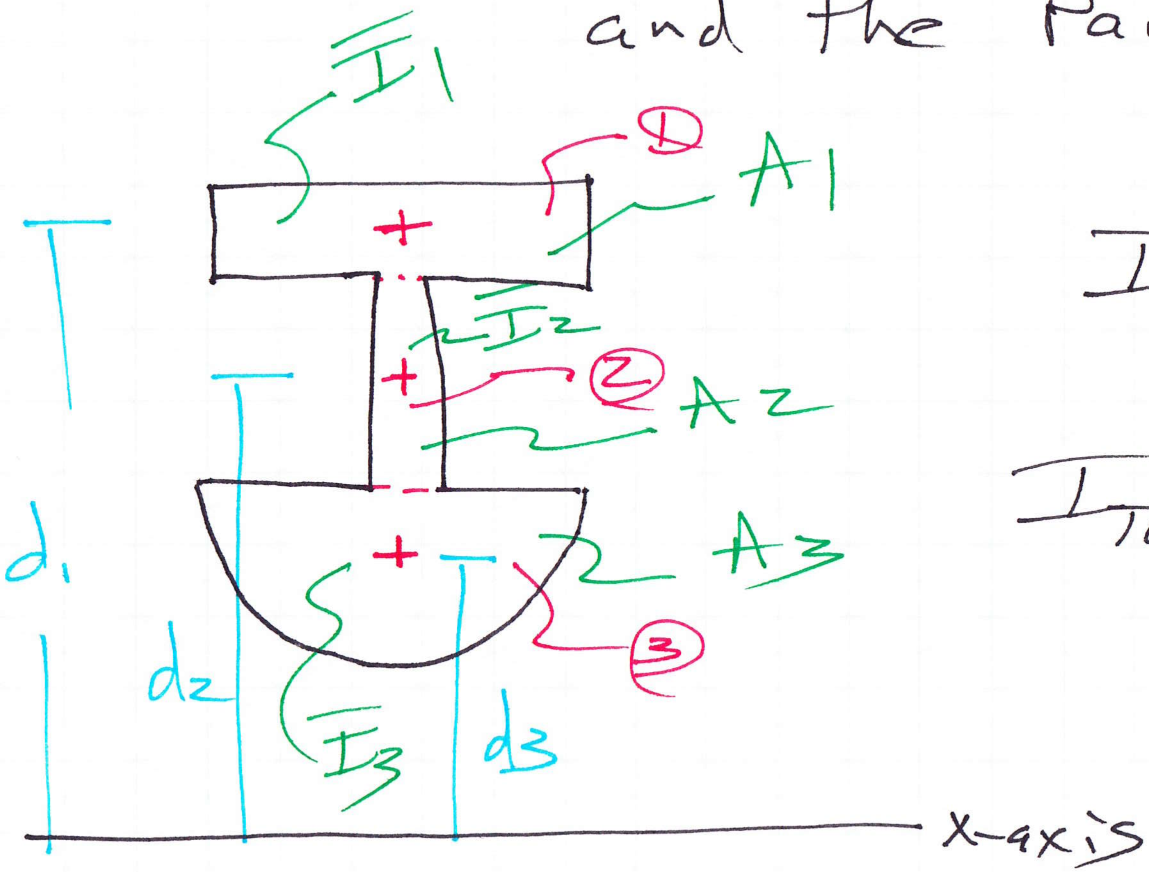
$$\frac{1}{8}\pi r^4 = \bar{I}_{x'} + \frac{\pi r^2}{2}\left(\frac{4r}{3\pi}\right)^2$$

$$\frac{\pi r^4}{8} = \bar{I}_{x'} + \left(\frac{\pi r^2}{2}\right)\left(\frac{16 r^2}{9\pi^2}\right)$$

$$\bar{I}_{x'} = \frac{\pi r^4}{8} - \frac{8}{9}\frac{r^4}{\pi}$$

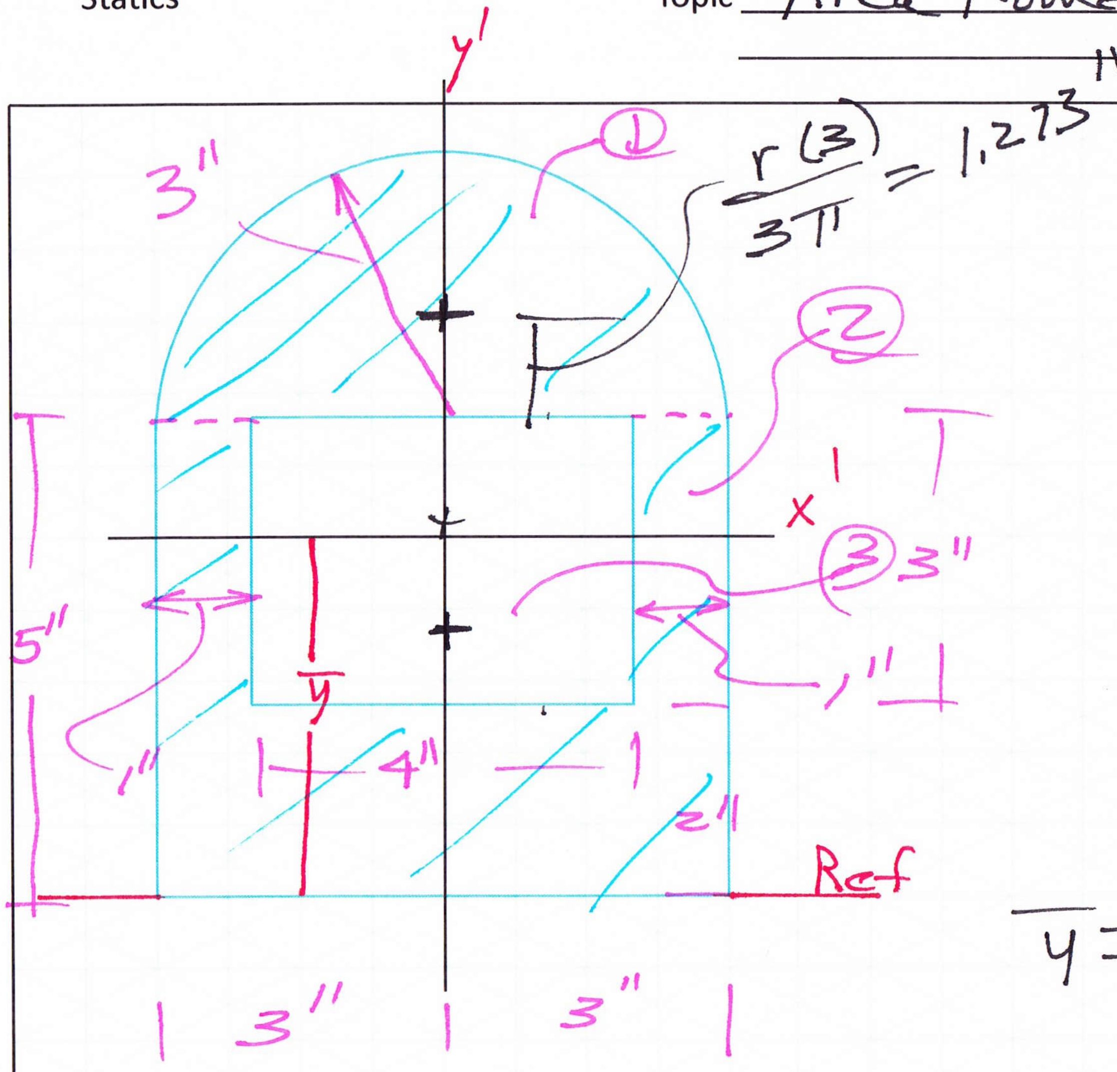
$$\bar{I}_{x'} = 0.1098\, r^4$$

Moments of Inertia using Composite Areas and the Parallel Axis Theorem.



$$I_{total} = \Sigma\left(\bar{I}_i + A_i d_i^2\right)$$

$$I_{Total} = \bar{I}_1 + A_1 d_1^2 + \bar{I}_2 + A_2 d_2^2 + \bar{I}_3 + A_3 d_3^2$$



Area Moments of Inertia about the horizontal and vertical centroidal axes.

$$A_1 = \frac{\pi}{2}(3)^2 = 14.14 \text{ in}^2$$

$$A_2 = 6(5) = 30 \text{ in}^2$$

$$A_3 = -(4)(3) = -12 \text{ in}^2$$

$$\bar{y} = \frac{(14.14)(6.273) + (30)(2.5) + (-12)(3.5)}{14.14 + 30 - 12}$$

$$\bar{y} = \frac{12.17}{32.14} = \underline{3.787 \text{ in}}$$

Segment 1 – semicircle

$$\bar{I}_1 = 0.1098\, r^4 = .1098(3)^4 = \underline{8.894\ in^4}$$

Segment 2 – Large Rectangle

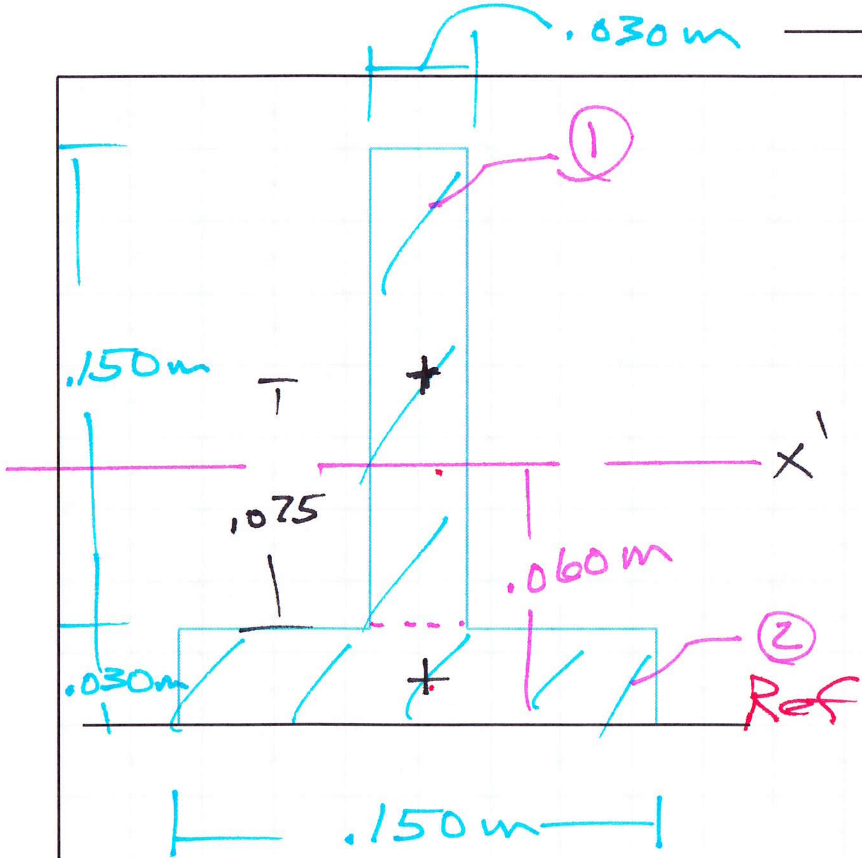
$$\bar{I}_2 = \frac{1}{12} b h^3 = \frac{1}{12}(6)(5)^3 = \underline{62.5\ in^4}$$

Segment 3 – Small Rectangle

$$\bar{I}_3 = \frac{1}{12} b h^3 = \frac{1}{12}(4)(3)^3 = \underline{9\ in^4} \quad \text{minus sign.}$$

$$\bar{I}_{x'} = \left[8.894 + 14.14(6.273 - 3.787)^2\right] + \left[62.5 + 30(2.5 - 3.787)^2\right] - \left[9 + 12(3.5 - 3.787)^2\right]$$

$$I_{x'} = 96.28 + 112.19 + (-9.988) = \boxed{198.5\ in^4}$$



Area Moment of Inertia about the horizontal Centroidal Axis

Centroidal Axis

$$\bar{Y} = \frac{(.030)(.150)(.105) + (.150)(.030)(.015)}{(.030)(.150) + (.150)(.030)}$$

$$\bar{Y} = 60.00 \times 10^{-3}\ m$$

Parallel Axis Theorem

$$I_{x'} = \sum\left(\bar{I}_i + A_i d_i^2\right)$$

Section	A_i	y_i	$A_i y_i$	$\bar{I}_x$	d_i	$A_i d_i^2$
1	$(.030)(.150)$ 4.50×10^{-3}	$.105$	472.5×10^{-6}	$\frac{1}{12}(.030)(.150)^3$ 8.438×10^{-6}	$(.105-.060)$ 45×10^{-3}	$(4.50\times10^{-3})(45\times10^{-3})^2$ 9.113×10^{-6}
2	$(.150)(.030)$ 4.50×10^{-3}	$.015$	67.50×10^{-6}	$\frac{1}{12}(.150)(.030)^3$ 337.5×10^{-6}	$(.015-.060)$ -45×10^{-3}	$(4.50\times10^{-3})(-45\times10^{-3})^2$ 9.113×10^{-6}
	$9.00\times10^{-3}\ m^2$		$540\times10^{-6}\ m^3$	8.775×10^{-6}		18.23×10^{-6}

$$\bar{y} = \frac{540\times10^{-6}}{9.00\times10^{-3}} = 60.00\times10^{-3}\ m$$

$$I_x' = 8.775\times10^{-6} + 18.23\times10^{-6}$$

$$I_x' = 27.00\times10^{-6}\ m^4$$

or

$$27.00\times10^{6}\ mm^4$$

Steps for Calculating Moment of Inertia Using Composite Areas

1) divide body into known sections
2) determine the A_i for each section
3) for a centroidal Moment of Inertia determine centroid
4) Determine $\bar{I}_i$ for each section
5) Determine d_i for each section
6) Plug into:

$$I_{total} = \Sigma \left(\bar{I}_i + A_i d_i^2 \right)$$